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Coplanar circles, quasi-affine invariance and calibration

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Abstract

We define the lines associated with two coplanar circles, and give the distributions of any two coplanar circles and their associated lines. Further we prove that the distribution of two coplanar circles with no real intersection and their associated lines is a quasi-affine invariance. Then the results are applied to calibrating a camera. The calibration method has the advantages: (1) it is based on conic fitting; (2) it does not need any matching. Experiments with two separate circles validate our quasi-affine invariance and show that the estimated camera intrinsic parameters are as good as those obtained by Zhang's (2000) method.
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Keywords: Associated lines with two coplanar circles; Projective invariance; Quasi-affine invariance; Circular points; Camera calibration

1. Introduction

A typical process of doing three-dimensional reconstruction of a scene is that a projective reconstruction is first found, and then is upgraded to a metric one. Maybe the projective reconstruction is split across the plane at infinity because an important piece of information is ignored—if the points are visible in the images, then they must have been in front of the cameras [5]. For this reason, the cheirality constraint on the reconstruction that the reconstructed points should lie in front of the cameras was proposed and has been used successfully in [1–5,7,9,10,12,13]. Hartley [3,5] gave the systematic theory of cheirality, and at the same time proposed the quasi-affine transformation, which lies part way between a projective and affine transformation. The reconstruction, up to a quasi-affine transformation, i.e. the quasi-affine reconstruction, is a projective reconstruction where the plane at infinity does not split across the scene.

In this paper, based on the quasi-affine invariance of a set of points in [3], we give a quasi-affine invariance associated with two coplanar circles, and then apply it to calibrating a camera. Extensive simulations and experiments with real images validate our quasi-affine invariant property and the calibrated intrinsic parameters are of high accuracy.

The paper is organized as follows. Some preliminaries, such as the quasi-affine transformations, the intrinsic parameters, and the circular points are introduced in Section 2. In Section 3, the lines associated with two coplanar circles are defined, and the distributions of the two circles and their associated lines are investigated, and further a quasi-affine invariance of two coplanar circles with no real intersection is proved. Then the results are applied to calibrating a camera in Section 4. In Section 5, calibration experiments are reported. Section 6 is some concluding remarks.

2. Preliminaries

In this paper, R^n denotes an n-dimensional Euclidean space, P^n an n-dimensional projective space. A bold capital letter denotes a matrix, and a bold lowercase letter denotes a column vector, which usually is a homogeneous vector. A matrix or a vector with τ denotes its transpose and the symbol '$\approx$' denotes the equality up to a scale.

A subset B of R^n is called *convex* if the line joining any two points in B also lies entirely within B. The convex hull of B, denoted $\bar{B}$, is the smallest convex set containing B. Let L_∞ be the infinity subspace in P^n. Hartley gave the definition of a quasi-affine transformation in [3]. For the convenience of later discussions, some of his results are recalled here under Definition 1, Theorem 1 and Proposition 1.

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Definition 1. Let B be a subset of R^n and h be a projective transformation of P^n. The projective transformation h is said to be *quasi-affine* with respect to the set B if $h^{-1}(L_\infty)$ does not meet $\bar{B}$.

Theorem 1. *If B is a point set in a plane (the object plane) in R^3 and B lies entirely in front of a projective camera, then the mapping from the object plane to the image plane defined by the camera is quasi-affine with respect to B.*

Proposition 1. *Let $\mathbf{x}_0$ and $\mathbf{x}_1$ be two points in space and π be a plane not passing through either of the points. Let h be a quasi-affine transformation with respect to the two points taking $\mathbf{x}_i$ to $\mathbf{x}_i'$ and mapping π to a plane π'. Then $\mathbf{x}_0$ and $\mathbf{x}_1$ lie on the same side of π if and only if $\mathbf{x}_0'$ and $\mathbf{x}_1'$ lie on the same side of π'.*

The two-dimensional Euclidean space R^2 and the line at infinity relative to it make up a two-dimensional projective space P^2. The three-dimensional Euclidean space R^3 and the plane at infinity relative to it make up a three-dimensional projective space P^3. We call the image of a line at infinity a *vanishing line*, the image of a point at infinity a *vanishing point*. The *absolute conic* is a virtual point conic in P^3 which is invariant to Euclidean transformations. It consists of points $\mathbf{x}=(x_1, x_2, x_3, 0)^\tau$ on the plane at infinity such that

$$x_1^2 + x_2^2 + x_3^2 = 0, \quad x_4 = 0,$$

or

$$\mathbf{x}^\tau \mathbf{x} = \mathbf{0}.$$

Every real plane π in R^3 intersects the plane at infinity at a real line l, which is the line at infinity relative to π. On the plane at infinity, l intersects the absolute conic at a pair of conjugate complex points, called the *circular points* relative to π (or of π). Every circle on the plane π passes through the two circular points.

The intrinsic parameters of a pinhole camera are

$$\mathbf{K} = \begin{pmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{pmatrix},$$

where f_u, f_v are the focal lengths along the image axes, s is the skew parameter, and (u_0, v_0) the principal point. The goal of calibrating a camera is to find the intrinsic parameters of the camera from images.

The equation of the image of the absolute conic is

$$\mathbf{x}^\tau \mathbf{K}^{-\tau} \mathbf{K}^{-1} \mathbf{x} = \mathbf{0},$$

which is a conic whose matrix dependents only on the intrinsic parameters $\mathbf{K}$. If $\mathbf{y}$ is the image of a circular point, then we have $\mathbf{y}^\tau \mathbf{K}^{-\tau} \mathbf{K}^{-1} \mathbf{y} = \mathbf{0}$.

3. A quasi-affine invariance of two coplanar circles

Definition 2. Two real circles C_1, C_2 on a Euclidean plane intersect at four points with multiplicity over complex field. Two of them are the circular points, which are conjugate complex and on the line at infinity. This line at infinity is denoted by L_∞. The other two intersections of C_1, C_2 are either a pair of conjugate complex, or real points, it follows that the line through them is always real, denoted by L. We call the two real lines L_∞, L the *lines associated with* C_1, C_2, or simply their *associated lines*, which are determined completely by C_1, C_2.

Totally there are six possible distributions for two coplanar circles, see Fig. 1. The distributions and the positions relative to their associated lines are, respectively:

(a) *The concentric case.* C_1, C_2 are virtually tangent at the two circular points on them (see [11]), the two associated lines all coincide to the line passing through the two common tangent points (i.e. the two circular points, so the line is the line at infinity).
(b) *The inner-tangent case.* C_1, C_2 intersect at a real point with multiplicity 2 and the two circular points, the associated lines are the unique common tangent line of the two circles, and the line at infinity that is separate from the two circles.
(c) *The outer-tangent case.* C_1, C_2 intersect at a real point with multiplicity 2 and the two circular points, the associated lines are a common tangent line of the two circles and the line at infinity that is separate from the two circles.
(d) *The intersecting case.* C_1, C_2 intersect at two real points and the two circular points, the two associated lines are the line through the two real intersections and the line at infinity that is separate from the two circles.

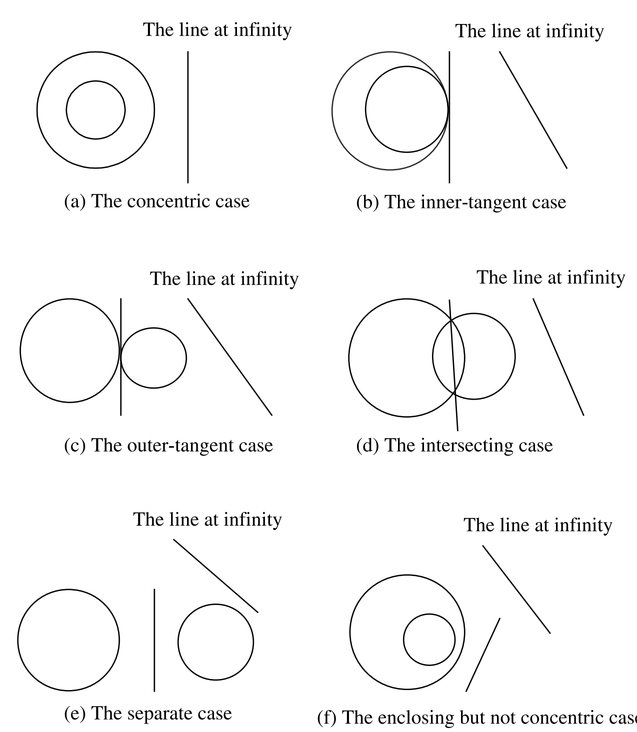

Fig. 1. All possible distributions of two coplanar circles and their associated lines.

(e) *The separate case.* C_1, C_2 intersect at two different pairs of conjugate complex, the two associated lines are a line such that the two circles lie on the different side of it, and the line at infinity such that the two circles lie on the same side of it. Proposition 2 will prove this.

(f) *The enclosing but not concentric case.* C_1, C_2 intersect at two different pairs of conjugate complex, and the two associated lines, one of which is the line at infinity, are both such that the two circles lie on the same side of them. Proposition 2 will prove this.

For each case of Cases (a)–(d), the relation given above between two coplanar circles and their two associated lines is unchanged under a real projective transformation. This is owing to the fact that a real projective transformation preserves the tangent properties, and transforms real intersections into real intersections, conjugate complex intersections into conjugate complex intersections. For each case of Cases (e) and (f), the relation given above between two coplanar circles and their two associated lines does not hold under a projective transformation, however we can prove it does keep unchanged under a quasi-affine transformation.

Proposition 2. *Let C_1, C_2 be two circles in R^2 that have no real intersection, L_∞ and L be their associated lines, where L_∞ is the line at infinity, then*

(i) *C_1, C_2 separate if and only if they lie on the different side of L, and lie on the same side of L_∞.*

(ii) *One circle is enclosed by the other one if and only if they lie on the same side of L_∞ and L.*

Furthermore, the above results are invariant under a quasi-affine transformation with respect to C_1 and C_2.

Proof.

(i) '⇒': If C_1, C_2 separate, then set up the Euclidean coordinate system as: one of the centers of C_1, C_2 as the origin O, the line through the two centers as the x-axis, the line through O and orthogonal to the x-axis as the y-axis, the radius of one of the circles as the unit length. For example, we take the coordinate system as in Fig. 2. Then the homogeneous equations of C_1, C_2 are, respectively

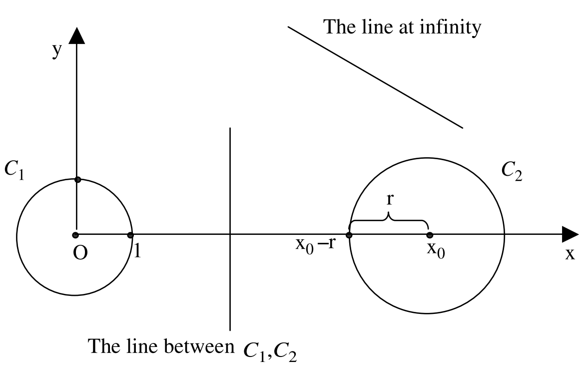

Fig. 2. Two separate circles.

$$x^2+y^2=z^2, \qquad (x-x_0z)^2+y^2=r^2z^2, \tag{1}$$

where r is the radius of C_2, x_0 horizontal coordinate of the center of C_2. Because C_1 and C_2 separate, $x_0>1+r$ and the four intersections of C_1 and C_2 are two pairs of conjugate complex. Solve Eq. (1) for x, y, z, and then compute the lines associated with C_1 and C_2, we have the two equations

$$L{:}x=\frac{x_0^2-r^2+1}{2x_0}, \quad L_\infty{:}z=0.$$

Because $x_0>1+r$, we can prove that the following inequality holds:

$$1<\frac{x_0^2-r^2+1}{2x_0}<x_0-r<\infty.$$

From the inequality, we know that C_1, C_2 lie on the different side of L, and lie on the same side of L_∞ as shown in Fig. 2.

In addition, the above proof is independent of the chosen Euclidean coordinate system. This is because if we set up another Euclidean coordinate system with the same or different unit length as the above one, then one of them can be transformed into another by a Euclidean transformation, or a similarity transformation, which preserves the line at infinity and preserves the relative position of objects.

'⇐': C_1 lies on one side of L and has no real intersection with L, C_2 lies on the other side of L and has no real intersection with L either, so C_1 and C_2 have no real intersection and are separate at finity. On the other hand, C_1, C_2 have no real intersection with L_∞, thus they are also separate at infinity. It follows that C_1, C_2 are separate.

(ii) It can be inferred directly by (i).

Induce Proposition 1 onto a plane and we have the result: let $\mathbf{x}_0$ and $\mathbf{x}_1$ be two points in a plane and l be a line passing through neither of the two points. Let h be a quasi-affine transformation with respect to the two points taking $\mathbf{x}_i$ to $\mathbf{x}_i'$ and mapping l to a line l'. Then $\mathbf{x}_0$ and $\mathbf{x}_1$ lie on the same side of l if and only if $\mathbf{x}_0'$ and $\mathbf{x}_1'$ lie on the same side of l'. Apply the result to any two points on C_1 or on C_2, or one point on C_1 and another point on C_2, and the two lines associated with C_1, C_2, we know that (i) and (ii) are invariant under a quasi-affine transformation with respect to C_1, C_2. □

Remark. For Cases (a)–(d), we call the distribution of the two coplanar circles and their associated lines a projective invariance, this is because it keeps unchanged under a projective transformation. For Cases (e) and (f), we call the distribution a quasi-affine invariance because it keeps unchanged under a quasi-affine transformation but not a projective transformation.

Theorem 2. *Let c_1, c_2 be the images of two coplanar circles C_1, C_2 under a pinhole camera, and let l and l_∞ be the images of the two associated lines of C_1 and C_2, which are determined by the intersections of c_1 with c_2, where l_∞ is the vanishing line, then:*

(i) *if C_1, C_2 separate, then c_1, c_2 lie on the different side of l and the same side of l_∞*;

(ii) *if one of C_1, C_2 is enclosed by the other, then both c_1, c_2 lie on the same side of l and l_∞*.

Proof. Let B be the point set of C_1 and C_2 (or parts of C_1 and C_2) in front of the camera, then B is projected to c_1, c_2. According to Theorem 1 in Section 2, we know that the projectivity from B to the image plane is quasi-affine with respect to B. Further by Proposition 2 and the result induced by Proposition 1 onto a plane, we know that the theorem is correct. □

The relation of two coplanar circles with their associated lines can be used to recognize the two circles' distribution and can be used in metrology. In addition, it can also be applied to calibrating a camera, which will be elaborated in Section 4.

4. Camera calibration using coplanar circles

Due to the special property that all circles pass through circular points, circles are very useful entities for calibrating cameras. In this section, we apply the projective and the quasi-affine invariance given in Section 3 to calibrating a camera. The main step is to find the images of circular points from the images of circles, which in turn give rise to constraints on the intrinsic parameters.

Assume there are two coplanar circles C_1, C_2 in the scene. Let c_1, c_2 be their images, e_1, e_2 be the algebraic homogeneous conic equations of c_1, c_2. Note that c_1, c_2 may be part of the geometric loci of e_1, e_2.

For Case (f), i.e. the enclosing but not concentric case, according to Theorem 2, we cannot identify the vanishing line from the two lines passing through, respectively, the two pairs of conjugate complex intersections of e_1, e_2. So we cannot find the images of circular points. But if the image of one center of C_1 or C_2, or a vanishing point of the plane containing the circles is known, the problem can be solved. This is because the center of a circle is in polarity with the circular points with respect to the circle, and a vanishing point is collinear with the images of circular points. The relative work has been given by Meng, Li and Hu [8].

For Cases (a)–(e), we can identify the vanishing line of the plane containing the circles by only using e_1, e_2 via the projective or quasi-affine invariance given in Section 3. More specially, the images of the circular points can be obtained by the fact that they are just on the vanishing line. The reason that we use the outlines of two circles but not a circle and its center to find the images of circular points is that using the outlines is more stable and usually circles or parts of circles in the scene are common but their centers are not.

From the above analysis, we know that if there are two coplanar circles in the scene, then we can find out two points (i.e. the images of two circular points) on the image of the absolute conic, it follows that we will have two linear constraint

Table 1
The estimated intrinsic parameters under different noise levels

Noise levels	f_u	f_v	s	u_0	v_0
0	1200.0	1000.0	2.0944	500.0	500.0
0.4	1200.2052	1000.2205	1.8704	499.5171	499.8348
0.8	1199.5839	999.2368	1.8498	499.9008	499.3187
1.2	1198.7646	998.0441	1.4266	500.05179	497.2479
1.6	1197.5775	996.5720	1.8971	500.4619	496.4293
2.0	1194.9216	994.4452	1.4309	500.6284	495.5680

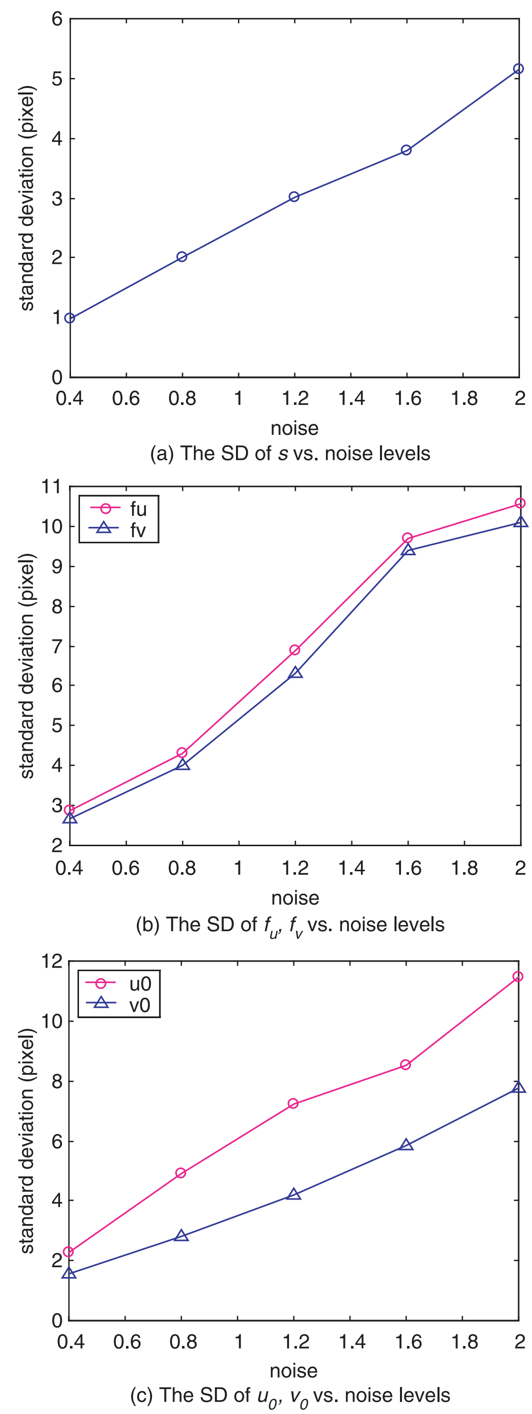

Fig. 3. The standard deviations of the estimated intrinsic parameters vs. noise levels: (a) of s; (b) of f_u, f_v; (c) of u_0, v_0.

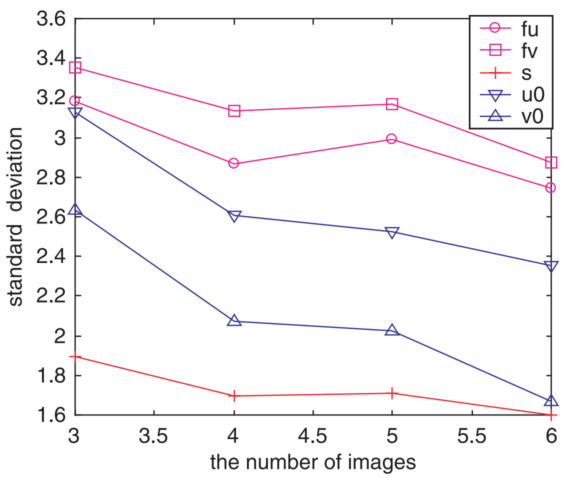

Fig. 4. The standard deviations of the estimated intrinsic parameters vs. the number of images.

equations on the intrinsic parameters. Assumed that the intrinsic parameters do not change, the camera can be calibrated by using three images of two coplanar circles. The motion between any two cameras should not be a pure translation, otherwise the equations will be dependent. One advantage of this calibration method is that neither matching between images nor matching between scene and images is needed.

The implementation of the proposed method for calibrating a camera is summarized as follows

Step 1 Fit the images c_1, c_2 of two space coplanar circles C_1, C_2 to obtain the homogeneous algebraic conic equations e_1, e_2.

Step 2 Solve the equation system e_1, e_2. According to the distribution of C_1, C_2, find the images of the circular points by the above discussions and denote the results as $\mathbf{y}_i$, $i=1,\ldots,n$, $n\geq 6$.

Step 3 Set up the constraint equations for the intrinsic parameters: $\mathbf{y}_i^\tau \mathbf{K}^{-\tau}\mathbf{K}^{-1}\mathbf{y}_i=0$, $i=1,\ldots,n$. And let $\mathbf{K}^{-\tau}\mathbf{K}^{-1}=\mathbf{N}$, then $\mathbf{N}$ is a symmetric matrix with five independent parameters. Regarding $\mathbf{N}$ as unknowns, find the least squares solution of the linear equations: $\mathbf{y}_i^\tau \mathbf{N}\mathbf{y}_i=0$, $i=1,\ldots,n$.

Step 4 Do Cholesky decomposition for $\mathbf{N}$, then inverse the result, we obtain an initial estimation of the intrinsic parameters, denoted as $\mathbf{K}_0$.

Step 5 Do non-linear optimization for the intrinsic parameters. Regarding $\mathbf{K}$ as unknowns, find the solution such that the following non-linear cost function is minimal $\sum_{i=1}^{n} ||\mathbf{y}_i^\tau \mathbf{K}^{-\tau}\mathbf{K}^{-1}\mathbf{y}_i||^2$ with $\mathbf{K}_0$ as the initial. The result is what we look for.

The proposed method in this section to compute the images of circular points from the images of two coplanar circles can also be extended to the case from the images of any two circles C_1, C_2 on two parallel planes π_1, π_2 by the following analysis: Because π_1 is parallel to π_2, the intersection of π_1 and the cone through C_2 with the camera's optical center as vertex is also a circle, denoted C_2'. The images of C_1, C_2 are also the images of the coplanar circles C_1, C_2' that have the same distribution as that of C_1, C_2, so we can also compute the images of circular points. Details

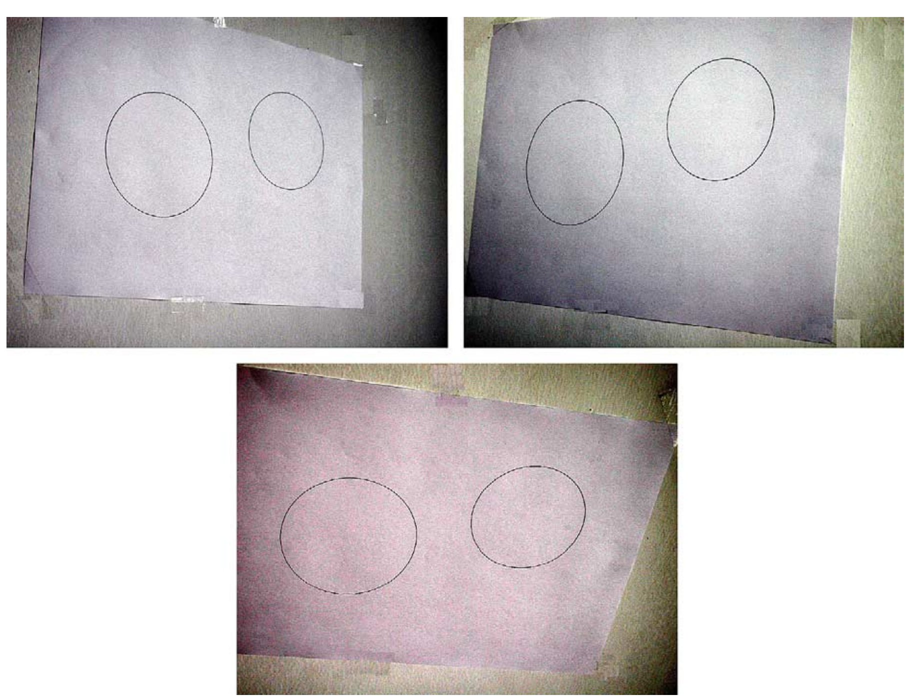

Fig. 5. The three images of two coplanar circles.

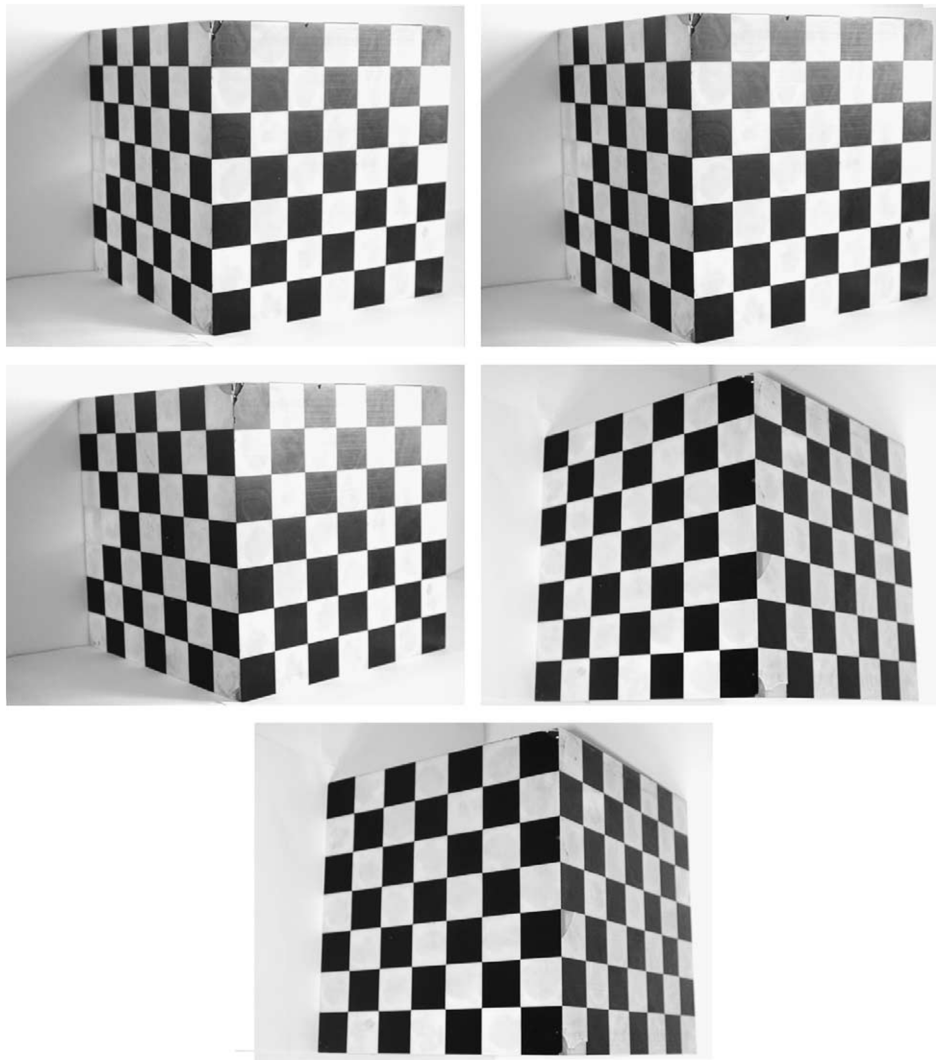

Fig. 6. Five images of a calibration grid for Zhang's calibration method.

can be found in [14] and the theory has been applied in [15]. We believe that it will have more applications. For example, in [6], the authors gave an approach for recovering 3D geometry from an image sequence of a single axis motion by fitting conics. They also use the images of circles to compute the images of circular points , if the images of two circles are separate then they cannot find the images of circular points and have to use the images of three circles. While, they can do it only from the images of two circles if the quasi-affine invariance proposed by us are applied.

5. Experiments

In order to verify the quasi-affine invariance introduced in this paper, some experiments with synthetic data and real images are carried out by using two separate circles.

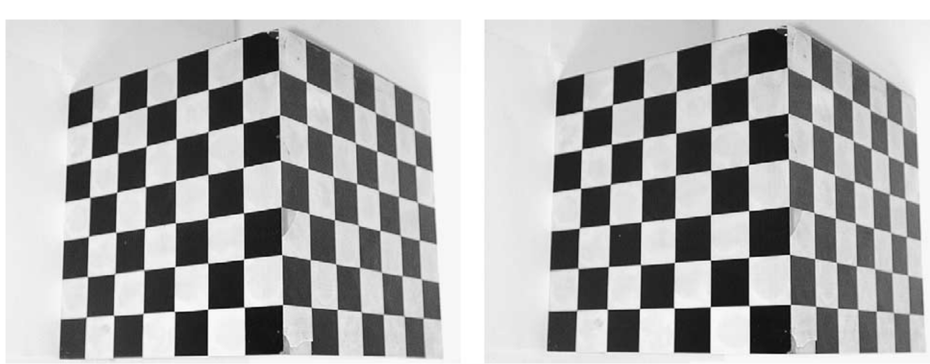

Fig. 7. Two images of a calibration grid for reconstruction.

Table 2
The estimated angles of two reconstructed orthogonal planes

	Using our calibration method	Using Zhang's calibration method
Fig. 7	89.7497°	89.5750°
Fig. 8	88.2654°	88.7187°

Table 3
The averages of the estimated angles between reconstructed parallel lines

	Using our calibration method	Using Zhang's calibration method
Fig. 7	0.989088°	0.989328°
Fig. 8	4.94641°	4.95762°

5.1. Experiments with synthetic data

In the experiments, the simulated camera has the following intrinsic parameters:

$$\mathbf{K} = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1200 & 2.0944 & 500 \\ 0 & 1000 & 500 \\ 0 & 0 & 1 \end{bmatrix}.$$

Take two circles in a three-dimensional plane, and then project them to the simulated image planes at three different positions. The images are of 1000×1000 pixels. Gaussian noise with mean 0 and standard deviation ranging from 0 to 2.0 pixels is added to the image points of the two circles, and then the intrinsic parameters are computed. For each noise level, we perform 100 times independent experiments, and the averaged results without optimization (the final optimization in fact made little improvement on the accuracy) are shown in Table 1. We also compute the variances of intrinsic parameters under different noise levels, the results are plotted in Fig. 3a–c.

In order to assess the performance of our calibration technique with the number of images, the calibrations using 4, 5, 6 images are performed, and the standard deviations of the estimated intrinsic parameters are shown in Fig. 4, where the added noise level is 0.5 pixels. It is clear that the errors tend to decrease with the number of the images increasing.

5.2. Experiments with real images

We use a CCD camera to take three photos of two coplanar circles as shown in Fig. 5, and the images are of 1024×768 pixels. Applied our calibration method to the three images, the estimated intrinsic parameters with final optimization are:

$$\mathbf{K}_1 = \begin{bmatrix} 1890.6198 & 2.6874 & 478.9313 \\ 0 & 1892.0379 & 372.9018 \\ 0 & 0 & 1 \end{bmatrix}.$$

To verify the estimated intrinsic parameters, the method proposed by Zhang [16] is used to calibrate the same camera

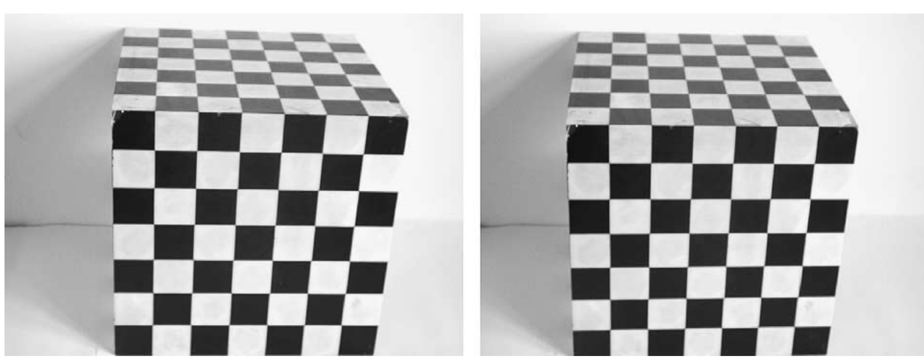

Fig. 8. Two images of a calibration grid for reconstruction.

Fig. 9. A pair of images used for the reconstruction of a terra cotta warrior's head.

Fig. 10. The reconstructed terra cotta warrior's head.

(the intrinsic parameters keep unchanged). The used images are shown in Fig. 6, and the calibration result is:

$$\mathbf{K}_2 = \begin{bmatrix} 1901.22 & -1.42336 & 522.775 \\ 0 & 1898.77 & 377.643 \\ 0 & 0 & 1 \end{bmatrix}.$$

$\mathbf{K}_1$, $\mathbf{K}_2$ are evaluated as follows.

The estimated intrinsic parameters $\mathbf{K}_1$, $\mathbf{K}_2$ are used to reconstruct a calibration grid from a pair of images of the calibration grid as shown in Fig. 7. The angles between two reconstructed orthogonal planes are given in Table 2, both of which are close to the ground truth of 90°. Consider the group of horizontal parallel lines on the left plane of the calibration grid, then the angles between any pair of the reconstructed parallel lines are computed, and the average are given in Table 3, both of which are close to the ground truth of 0°. The process is repeated by another pair of images in Fig. 8, where the parallel lines considered are the horizontal parallel lines on the lower plane, the results are also shown in Tables 2 and 3. We can see that our method is as good as Zhang's method in [16].

Fig. 9 shows a pair of images used for the reconstruction of a terra cotta warrior's head. Fig. 10 is the reconstructed head by using our estimated intrinsic parameters $\mathbf{K}_1$. It is clear that the visual quality of the reconstructed head is satisfactory.

6. Conclusions

We have defined the lines associated with two coplanar circles, and given the distributions of any two coplanar circles and their associated lines. When two coplanar circles have no real intersection, we further proved that the relative position of these two circles and their associated lines is invariant under a quasi-affine transformation. Then, the results are applied to calibrating a camera. Experiments show that our method is as good as Zhang's method in [16]. Our calibration method has the advantages: (i) it is based on conic fitting; (ii) it does not need any matching. During our experiments, we found that the errors of estimated intrinsic parameters depend on the relative position between the camera and the circles, and how to find out robust configuration will be our further work.

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