Euclidean reconstruction of a circular truncated cone only from its uncalibrated contours

Yihong Wu *, Guanghui Wang, Fuchao Wu, Zhanyi Hu

National Laboratory of Pattern Recognition, Institute of Automation, Chinese Academy of Sciences, No. 95 EastRoad of ZhongGuangCun, P.O. Box 2728, Beijing 100080, People’s Republic of China

Received 30 January 2004; received in revised form 17 November 2005; accepted 16 February 2006

Abstract

We present a method to recover a circular truncated cone only from its contour up to a similarity transformation. First, we find the images of circular points, and then use them to calibrate the camera with constant intrinsic parameters from two or three contours of a circular truncated cone, or from a single contour of a circular truncated cylinder. Second, we give an analytical solution of the relative pose between each camera and the circular truncated cone. After the camera’s intrinsic parameters and pose are recovered, the circular truncated cone or cylinder is reconstructed from one image. Experiments on both simulated data and real images are performed and validate our approach.

Keywords: Circular truncated cone or cylinder; Absolute conic; Circular point; Camera intrinsic parameter; Camera extrinsic parameter; Euclidean reconstruction

1. Introduction

Recovering three-dimensional shape from images is one of the main tasks in computer vision. However, from only contours in images to recover three-dimensional shape is not an easy work since the contours in different images usually do not match. This results in the difficulty of using the stereo vision method [1,2] to find the camera’s intrinsic, extrinsic parameters and then the shape. Thus, from contours to shape, some methods in the past assume that either the intrinsic parameters or extrinsic parameters are known [3–7], or try to find the matching points [8,9], or require the camera to do specific motions [10–12], or only give a projective or affine reconstruction [11,13].

But, for a circular truncated cone or cylinder, we find that the camera’s intrinsic and extrinsic parameters can be directly recovered from its contours if the intrinsic parameters are kept unchanged. No matching or no specific camera motion is needed in our method, and the recovering is a Euclidean reconstruction. In addition, only from a single contour of a circular truncated cylinder, the recovering process is also possible if one constraint on the intrinsic parameters is available.

Our idea in this paper is just to find the images of circular points, then from them the intrinsic and extrinsic parameters are derived. In addition, the circles at the cone’s brims and the height of the cone are recovered, in other words, a Euclidean reconstruction of the circular truncated cone is obtained up to a similarity transformation. The intrinsic parameters are recovered based on a quasi-affine invariance proposed for parallel circles in [14], which is much different from the previous calibration methods from circles in [15–17].

The paper is organized as follows: some preliminaries are given in Section 2. In Section 3, the constraints on the intrinsic and extrinsic parameters are formulated. Then, the reconstruction method and an algorithm are elaborated in Sections 4 and 5, respectively. Experimental results are reported in Section 6.

2. Preliminaries

In this paper, we use ‘=’ to denote the equality up to a scalar, a bold letter to denote a matrix or a homogeneous vector.

Under a pinhole camera, a point \( \mathbf{X} \) in space is projected to a point \( \mathbf{x} \) in the image by

\[
\mathbf{x} = \mathbf{K} [\mathbf{R}, \mathbf{t}] \mathbf{X},
\]

(1)
where

\[ K = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \]

is the matrix of camera’s intrinsic parameters, and \( R, t \) are a 3 \times 3 rotation matrix and a three-dimensional translation vector.

The absolute conic consists of points \( X = (x_1, x_2, x_3, 0)^T \) at infinity such that

\[ x_1^2 + x_2^2 + x_3^2 = 0, \]

or:

\[ X^T X = 0. \]

Its image \( \omega \) is:

\[ x^T K^{-1} K^{-T} x = 0. \] (2)

Every real plane other than the plane at infinity in space intersects the absolute conic at a pair of conjugate complex points, called the circular points of the plane. Every real finite circle and the absolute conic have two intersection points, which are the two circular points of the plane containing this circle. Because two parallel planes have common points at infinity, their circular points coincide. The image of a point at infinity is called a vanishing point and the image of a line at infinity is called a vanishing line.

A circular truncated cone in this paper is a symmetric object with two circles as its brims, and has a vertex at which all its generators intersect, as shown in Fig. 1, where the third one is a circular truncated cylinder whose vertex is at infinity.

3.1. Constraints on the intrinsic parameters

(i) If \( c_1 \) and \( c_2 \) intersect at one or two real points, then they must intersect at another and unique pair of conjugate complex points, which are just the images of a pair of circular points.

(ii) If \( c_1 \) and \( c_2 \) are separate, then the intersection points of \( c_1 \) and \( c_2 \) are two pairs of conjugate complex points. Passing through each of these two pairs gives a real line. One of the two real lines lies between \( c_1 \) and \( c_2 \) denoted by \( l_{\text{inner}} \), and the other one does not denoted by \( l_{\text{outer}} \), as

\[ \text{The visible part of } c_1 \quad \text{The visible part of } c_1 \]

\[ \text{The visible part of } c_2 \quad \text{The visible part of } c_2 \]

Fig. 2. Contours of circular truncated cones.
shown in Fig. 3. When camera does not lie between the planes containing \( C_1 \) and \( C_2 \), the intersection points of \( c_1 \) and \( c_2 \) on \( l_{outer} \) are the images of a pair of circular points. Otherwise, the intersection points of \( c_1 \) and \( c_2 \) on \( l_{inner} \) are the images of a pair of circular points.

**Proof.** Because the planes containing \( C_1 \) and \( C_2 \) are parallel, the quadric cone with the camera optical center as its vertex and through \( C_1 \) intersects the plane containing \( C_2 \) at another circle \( C_3 \). \( C_2 \) and \( C_3 \) are two coplanar circles, which are just imaged into \( c_2 \) and \( c_1 \). Then, the part (i) of the theorem and the distribution of \( c_1 \), \( c_2 \), \( l_{inner} \), \( l_{outer} \) in part (ii) of the theorem are proved by the quasi-affine invariance in [18]. The location of the images of a pair of circular points in the part (ii) is proved by [14]. \( \square \)

After finding the images of circular points, according to projective geometric knowledge [19], and (2) we have the following theorem.

**Theorem 2.** Let \( z_1 \) and \( z_2 \) be the images of circular points determined by \( c_1 \) and \( c_2 \), then we have two independent constraints on the intrinsic parameters:

\[
z_1^T K^{-1} K^{-1} z_1 = 0, \quad z_2^T K^{-1} K^{-1} z_2 = 0.
\] (3)

Let \( v_\infty \) be the vanishing point of the line orthogonal to \( C_1 \) or \( C_2 \), and let \( c_1 \) and \( c_2 \) be the symmetric matrices of \( c_1 \) and \( c_2 \), then (\( s_1 \) is a scalar):

\[
K^{-T} K^{-1} v_\infty = s_1 z_1 \times z_2,
\] (4)

\[
(c_1^{-1} (z_1 \times z_2) \times c_2^{-1} (z_1 \times z_2)) \cdot v_\infty = 0.
\] (5)

The total number of independent constraints from (3)–(5) on \( K^{-T} K^{-1} \) after eliminating \( s_1 \) and \( v_\infty \) is three. If \( \Omega \) is a circular truncated cylinder, then \( v_\infty \) coincides with the intersection of \( l_1 \) and \( l_2 \), and the total number of independent constraints from (3)–(5) on \( K^{-T} K^{-1} \) after eliminating \( s_1 \) is four.

It follows that from the contour of \( \Omega \) in a single view, we can identify the images of a pair of circular points and establish the constraints on the intrinsic parameters by Theorems 1 and 2. If the camera’s intrinsic parameters are kept unchanged and the motions between cameras are not pure translations, the camera can be completely calibrated from two images (four linear constraints and two quadric constraints) and be linearly calibrated from three images (six linear constraints and three quadric constraints). Moreover, we only need one image to calibrate the camera completely if \( \Omega \) is a cylinder (four linear constraints from a contour) and one additional constraint on the intrinsic parameters is available.

### 3.2. Constraints on the extrinsic parameters

After the intrinsic parameters are solved, the extrinsic parameters or the relative position between the camera and the circular truncated cone or cylinder can be obtained as follows.

Consider any one view taken. Because circular points are at infinity, the line through their image points \( z_1 \) and \( z_2 \) is a vanishing line denoted as \( z_1 z_2 \). Let \( o \) be the pole of \( z_1 z_2 \) with respect to the ellipse \( c_1 \) in the image. Take a real point \( v_x \) on \( z_1 z_2 \), and then compute the harmonic point \( v_y \) of \( v_x \) on \( z_1 z_2 \) with respect to \( z_1 \) and \( z_2 \). We establish the world coordinate system as in the following: take the center \( O \) of \( C_1 \), whose image is \( o \), as the origin \((0 0 0 1)\). \( V_x \), whose image is \( v_x \), as the direction \((1,0,0,0)\) of \( X\)-axis. \( V_y \), whose image is \( v_y \), as the direction \((0,1,0,0)\) of \( Y\)-axis.

Then by (1) and letting \( r_1, r_2, r_3 \) be the three columns of the rotation matrix \( R \), we have:

\[
o = K[r_1 \ r_2 \ r_3 \ t](0 \ 0 \ 0 \ 1)^T = Kt,
\]

\[
v_x = K[r_1 \ r_2 \ r_3 \ t](1 \ 0 \ 0 \ 0)^T = Kr_1,
\]

\[
v_y = K[r_1 \ r_2 \ r_3 \ t](0 \ 1 \ 0 \ 0)^T = Kr_2.
\]

Thus:

\[
t \approx K^{-1} o, \quad r_1 = \frac{K^{-1} v_x}{\|K^{-1} v_x\|}, \quad r_2 = \frac{K^{-1} v_y}{\|K^{-1} v_y\|}.
\]

Then \( r_3 = r_1 \times r_2 \).

We cannot determine the absolute depth of the cone since no metric is involved during the calibration. In other words, the reconstruction can only be done up to a global scale. Taking this in mind, we always assume \( t = K^{-T} o \) in the following sections.

### 4. Euclidean reconstruction of a circular truncated cone

Let \( K \) be the intrinsic parameters calibrated by the methods in Section 3.1, choose one image among the acquired images, and let \( R \) and \( t \) be the extrinsic parameters associated to the chosen image recovered in Section 3.2, then the projection matrix associated to the image is \( P = K[r_1 \ r_2 \ r_3 \ t] = [K \ R \ t] \).

In the chosen image, let \( o_3 \) be the pole of \( z_1 z_2 \) with respect to the ellipse \( c_2 \). We are going to reconstruct the truncated cone from this image under the coordinate system as that when solving \( R \) and \( t \), i.e., the coordinate system whose origin is the center of \( C_1 \), whose \( Z\)-axis is the line through the centers of \( C_1 \) and \( C_2 \), and whose \( X\)-\( Y\) plane is the plane containing \( C_1 \). So the coordinates of the center of
C_2 is (0 0 Z_0 1)^T. Since o_2 is the image of the center of C_2, we have:

s_2 o_2 = P(0 0 Z_0 1)^T
= K[r_1 \ r_2 \ r_3 \ t](0 0 Z_0 1)^T
= Z_0 Kr_3 + K t. \hspace{1cm} (6)

There are three equations in (6), from which s_2 and Z_0 can be solved. Then the two space circles of C_1 and C_2 are reconstructed as the equations

\begin{align*}
C_1: & \left\{ \begin{array}{l}
(X \ Y \ Z \ 1)P^T c_1 P(X \ Y \ Z \ 1)^T = 0 \\
Z = 0
\end{array} \right. \\
C_2: & \left\{ \begin{array}{l}
(X \ Y \ Z \ 1)P^T c_2 P(X \ Y \ Z \ 1)^T = 0 \\
Z = Z_0
\end{array} \right.
\end{align*}

this is because for any point (X Y Z 1)^T on C_1 (C_2), its image P(X Y Z 1)^T is on the conic c_1 (c_2), and it is on the plane Z=0 (Z=Z_0).

After the two circles of the circular truncated cone are reconstructed, the circular truncated cone can be constructed completely. We use the line segment, which is coplanar with Z-axis and whose two ends are on the two circles, to fit the profile of the circular truncated cone.

5. Algorithm

An outline of the algorithm to reconstruct a circular truncated cone or cylinder from its contours is given as:

Step 1 Fit the contour to obtain c_1, c_2, l_1, l_2 in every view.
Step 2 Find the intrinsic parameters of the camera.
    Step 2.1 Find the images of circular points in every view by Theorem 1.
    Step 2.2 Set-up the constraints on \( A = K^{-T}K^{-1} \) in every view by Theorem 2.
    Step 2.3 Solve all the constraint equations in Step 2.2 for A, then do Cholesky decomposition to obtain K. The solving method is detailed as: if we have five or more than five linear equations, solve the intrinsic parameters from these linear equations by least-square method to obtain initial values of A. Otherwise, combine linear equations with prior knowledge of camera intrinsic parameters to obtain initial values of A, or do polynomial eliminations to obtain univariate quadratic equations on which a quasi-least-square method is employed to obtain initial values of A. After obtaining the initial values, we do optimization for A by using the sum of the residual squares of all linear and nonlinear equations as cost function.

Step 3 Choose one view, find the relative position between the camera and the cone or cylinder by the method given in Section 3.2.
Step 4 Reconstruct the circular truncated cone or cylinder by the method given in Section 4.

6. Experiments

6.1. Simulations

The simulated camera has the following intrinsic parameters:

\[ K = \begin{bmatrix}
  f_u & 0 & u_0 \\
  0 & f_v & v_0 \\
  0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
  1500 & 0 & 380 \\
  0 & 1300 & 380 \\
  0 & 0 & 1
\end{bmatrix}. \]

The simulated circular truncated cylinder is:

\begin{align*}
C_1: & \left\{ \begin{array}{l}
X^2 + Y^2 = 20^2 \\
Z = 0
\end{array} \right. \\
C_2: & \left\{ \begin{array}{l}
X^2 + Y^2 = 20^2 \\
Z = 40
\end{array} \right.
\end{align*}

We are going to reconstruct the cylinder from a single image assuming that the intrinsic parameter s=0 is known.

One image is generated and random noise (noise unit: pixel) with different noise levels is added to it (Suppose \((u,v)^T\) is an image point, then the noise with noise level \( \Delta \) is added by: \((u,v)^T + \Delta rand, rand)^T\), where \( rand \) is a random value within \((-0.5, 0.5)\). At each noise level, 100 trials are done, and the averages of the recovered results are shown in Tables 1–4. We also give the similarity invariants—the ratios of the estimated heights to the ground truth 2 = 40/20.

The standard deviations of the estimated intrinsic parameters are plotted in Fig. 4. We also plot the standard deviations of the estimated heights, of the estimated radii, and of the ratios of the estimated heights to the estimated

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Averages of the estimated intrinsic parameters vs. noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise levels</td>
<td>0.0</td>
</tr>
<tr>
<td>( f_u )</td>
<td>1500.000</td>
</tr>
<tr>
<td>( f_v )</td>
<td>1300.000</td>
</tr>
<tr>
<td>( u_0 )</td>
<td>500.000</td>
</tr>
<tr>
<td>( v_0 )</td>
<td>380.000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Averages of the reconstructed heights of the cylinder vs. noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>Noise levels</td>
<td>0.0</td>
</tr>
</tbody>
</table>
radii in Fig. 5. We can see that the deviations of the two recovered radii are quite close, and the deviations of the ratios of the estimated height to the two radii are also quite close.

The above results show that our algorithm is stable to noise. The main reason is that our algorithm is based on fitting conics and lines.

6.2. Experiments on real data

6.2.1. Using a single image from a cylinder

The image is captured by a Nikon COOLPIX990 camera. For this camera, the aspect ratio is 1, i.e. $f_u=f_v$. This prior knowledge results in a constraint on $A=K^{-1}K^{-1}$, as (see [20]):

$$A_{22} \left( \frac{A_{12}}{A_{11}} \right)^2 = 1. \quad (7)$$

Then for a circular truncated cylinder, by Theorem 2 and (7), $A$ has five constraints from a single image. Therefore, $A$ or $K$ can be solved out completely from a single image. Thus, a circular truncated cylinder can be reconstructed from a single image by our method.

Table 3
Averages of the coefficients of the reconstructed circles $C_1$ vs. noise

<table>
<thead>
<tr>
<th>Noise levels</th>
<th>0.0</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>coeff($Y^2$)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>coeff($XY$)</td>
<td>$1.209 \times 10^{-16}$</td>
<td>$7.434 \times 10^{-18}$</td>
<td>$1.242 \times 10^{-17}$</td>
<td>$7.110 \times 10^{-18}$</td>
<td>$5.197 \times 10^{-18}$</td>
<td>$8.221 \times 10^{-18}$</td>
</tr>
<tr>
<td>coeff($X$)</td>
<td>$1.890 \times 10^{-15}$</td>
<td>$2.609 \times 10^{-16}$</td>
<td>$6.097 \times 10^{-16}$</td>
<td>$9.929 \times 10^{-16}$</td>
<td>$9.917 \times 10^{-17}$</td>
<td>$3.247 \times 10^{-16}$</td>
</tr>
<tr>
<td>coeff($Y$)</td>
<td>$5.539 \times 10^{-15}$</td>
<td>$2.737 \times 10^{-16}$</td>
<td>$8.461 \times 10^{-17}$</td>
<td>$7.539 \times 10^{-16}$</td>
<td>$6.209 \times 10^{-16}$</td>
<td>$2.009 \times 10^{-16}$</td>
</tr>
<tr>
<td>coeff(const)</td>
<td>$-25.000$</td>
<td>$-25.028$</td>
<td>$-25.017$</td>
<td>$-25.292$</td>
<td>$-25.447$</td>
<td>$-25.652$</td>
</tr>
</tbody>
</table>

(coeff means ‘coefficient’ and we have made the coefficient of $X^2$ as 1).

Table 4
Averages of the coefficients of the reconstructed circles $C_2$ vs. noise

<table>
<thead>
<tr>
<th>Noise levels</th>
<th>0.0</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>coeff($Y^2$)</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>coeff($XY$)</td>
<td>$1.498 \times 10^{-16}$</td>
<td>$1.082 \times 10^{-17}$</td>
<td>$1.502 \times 10^{-17}$</td>
<td>$1.475 \times 10^{-18}$</td>
<td>$6.004 \times 10^{-19}$</td>
<td>$8.339 \times 10^{-18}$</td>
</tr>
<tr>
<td>coeff($X$)</td>
<td>$-1.624 \times 10^{-8}$</td>
<td>$7.055 \times 10^{-5}$</td>
<td>$1.117 \times 10^{-4}$</td>
<td>$-1.848 \times 10^{-4}$</td>
<td>$2.479 \times 10^{-4}$</td>
<td>$-6.404 \times 10^{-4}$</td>
</tr>
<tr>
<td>coeff($Y$)</td>
<td>$1.180 \times 10^{-9}$</td>
<td>$-5.058 \times 10^{-6}$</td>
<td>$-6.877 \times 10^{-6}$</td>
<td>$1.622 \times 10^{-5}$</td>
<td>$-1.344 \times 10^{-5}$</td>
<td>$5.556 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

(coeff means ‘coefficient’ and we have made the coefficient of $X^2$ as 1).

Table 5
Averages of the ratios of the reconstructed heights to the reconstructed radii vs. noise

<table>
<thead>
<tr>
<th>Noise levels</th>
<th>0.0</th>
<th>0.2</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ratios w.r.t. $C_1$</td>
<td>2.0000</td>
<td>1.9990</td>
<td>2.0001</td>
<td>1.9914</td>
<td>1.9892</td>
<td>1.9861</td>
</tr>
<tr>
<td>Ratios w.r.t. $C_2$</td>
<td>2.0000</td>
<td>1.9990</td>
<td>2.0002</td>
<td>1.9914</td>
<td>1.9893</td>
<td>1.9862</td>
</tr>
</tbody>
</table>

Fig. 4. SD of the estimated intrinsic parameters vs. noise.

Fig. 5. SD of the estimated radii, heights and the ratios of the heights to the estimated radii vs. noise.
The used image is from a water cup as shown in Fig. 6 with the size of 1024×768 pixels. The pixels marked by white color are extracted by Canny edge detector (see Fig. 7), then are fitted to two conics and two lines by the least square method. By employing the proposed algorithm in Section 5, two groups of intrinsic parameters are obtained as:

\[
K = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2887.3772 & -8.2528 & 517.7515 \\ 0 & 2887.3772 & 194.7500 \\ 0 & 0 & 1 \end{bmatrix},
\]

\[
K_1 = \begin{bmatrix} f_u & s & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 95.7285 & 3688.7492 & 583.9487 \\ 0 & 95.7285 & -2104.5338 \\ 0 & 0 & 1 \end{bmatrix},
\]

Discard the unreasonable \(K_1\). From \(K\), the reconstructed circles at the brims of the cup are:
They are nearly two circles. The radii of the two reconstructed circles and the reconstructed height of the cup are:

\[ r_1 = 0.7440, \quad r_2 = 0.7369, \quad \text{height} = 1.6917. \]

From the results the similarity invariants—the ratios of the reconstructed height to the reconstructed radii are computed as:

\[ \text{ratio}_{hr_1} = 2.2737, \quad \text{ratio}_{hr_2} = 2.2958. \]

They are both close to the ground truth 2.3171 = 9.5/4.1.

The reconstructed results with texture mapping under different viewpoints are shown in Fig. 8.

6.2.2. Using three images from a cone

We use three images as shown in Fig. 9 to linearly calibrate a camera. The images are taken by FUJIFILM MX-2900 digital camera and are of size 640×480 pixels.

For every image in Fig. 9, the pixels marked by white color as shown in Fig. 10 are extracted by Canny edge detector, and then are fitted to two conics \( c_1, c_2 \) and two lines \( l_1, l_2 \). By applying the algorithm in Section 5, the intrinsic parameters are calibrated as:

\[
K = \begin{bmatrix}
    f_u & s & u_0 \\
    0 & f_v & v_0 \\
    0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
    1093.5 & -1.6 & 319.4 \\
    0 & 1116.2 & 239.7 \\
    0 & 0 & 1
\end{bmatrix},
\]

and the extrinsic parameters of the camera relative to the first image are estimated as:

Fig. 9. Used three images.

Fig. 10. Extracted parts in the image.
Then, the reconstruction of $c_1$ and $c_2$ are, respectively:

\[
\begin{align*}
C_1: & \quad \left\{ \begin{array}{l}
X^2 + Y^2 + 24.76X + 5.86Y + 124.1497 = (X + 12.38)^2 + (Y + 2.93)^2 - 6.14^2 = 0 \\
Z = 22.80
\end{array} \right. \\
C_2: & \quad \left\{ \begin{array}{l}
X^2 + Y^2 + 24.22X + 6.30Y + 108.4110 = (X + 12.11)^2 + (Y + 3.15)^2 - 6.94^2 = 0 \\
Z = 0
\end{array} \right.
\]

It is clear that the distance between the two recovered circles is 22.80. We compute the ratio of the recovered radius of $C_1$ to the distance, the ratio of the recovered radius of $C_2$ to the distance, and obtain:

\[
\text{ratio}_{r_1d} = \frac{6.14}{22.8} = 0.2693,  \quad \text{(8)}
\]

\[
\text{ratio}_{r_2d} = \frac{6.94}{22.8} = 0.3044
\]

The ground truths of the radii of $C_1$, $C_2$ and the distance between the two circles are: 59.2, 66.5 and 216.5 mm, respectively. From the truths, the corresponding true ratios of (8) are:

\[
\text{ratio}_{r_1d} = \frac{59.2}{216.5} = 0.2734,  \quad \text{(9)}
\]

\[
\text{ratio}_{r_2d} = \frac{66.5}{216.5} = 0.3072
\]

We can see that the values of (8) and (9) are very close, showing that our algorithm is valid and of higher accuracy.

The reconstructed results with texture mapping under different viewpoints are shown in Fig. 11.
7. Conclusions

We have presented an algorithm to reconstruct a circular truncated cone only from its uncalibrated contours. A circular truncated cone can be reconstructed up to a similarity transformation from its uncalibrated contours in two views and the reconstruction can be linear from the contours in three views. Moreover, if the cone is a circular truncated cylinder and one additional constraint on the intrinsic parameters is available, the reconstruction is possible from only a single uncalibrated contour. Simulations and experiments on real data were performed showing the correctness and stability of our method.

Acknowledgements

The work is supported by the National Key Basic Research and Development Program (973) under grant no. 2002CB312104 and the National Natural Science Foundation of China under grant no. 60475009.

References