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# Accurate, Dense, and Robust Multi-View Stereopsis (PMVS)

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# Abstract

This article proposes a novel algorithm for multiview stereopsis that outputs a dense set of small rectangular patches covering the surfaces visible in the images. Stereopsis is implemented as a **match, expand, and filter procedure**, starting from a sparse set of matched keypoints, and repeatedly expanding these before using visibility constraints to filter away false matches. Simple but effective methods are also proposed to turn the resulting patch model into a mesh which can be further refined by an algorithm that enforces both photometric consistency and regularization constraints. A quantitative evaluation on the Middlebury benchmark shows that the proposed method outperforms all others submitted so far for four out of the six datasets.

# Key procedures of PMVS

## (1) *Feature detection and matching:*

features found by Harris and Difference-of-Gaussians operators are matched across multiple pictures, yielding a sparse set of patches.

The procedure (2) and (3) iterative 3 times

## (2) *Expansion:*

a technique is used to spread the initial matches to nearby pixels and obtain a dense set of patches.

## (3) *Filtering:*

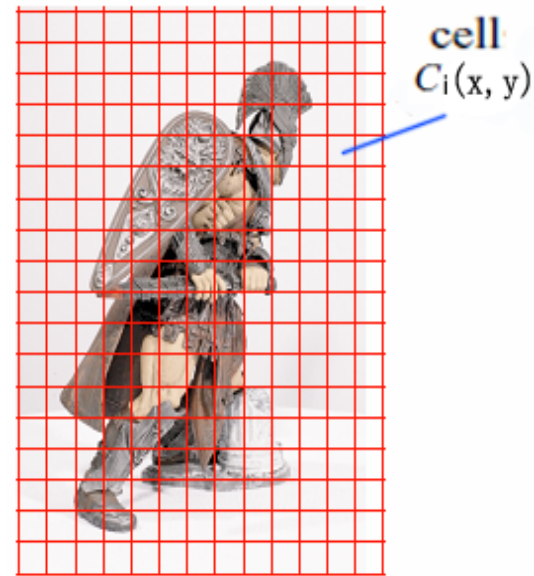
visibility constraints are used to eliminate incorrect matches lying either in front or behind the observed surface.

## (4) *Polygonal surface reconstruction*



# Some definitions

- **Cell:** we associate with each image a regular grid of  $\beta_1 \times \beta_1$  pixels cells  $C_i(x, y)$
- **$V(p)$  and  $V^*(p)$ :**  $V(p)$  denote a set of images in which patch  $p$  is visible.  
$$V^*(p) = \{I | I \in V(p), h(p, I, R(p)) \leq \alpha\}$$
- **$Q_i(x, y)$  and  $Q^*_i(x, y)$ :** given a patch  $p$  and its visible images  $V(p)$ , we simply project  $p$  into each image in  $V(p)$  to identify the corresponding cell. Then, each cell  $C_i(x, y)$  remembers the set of patches  $Q_i(x, y)$  that project into it. Similarly, we use  $Q^*_i(x, y)$  to denote the patches that are obtained by the same procedure, but with  $V^*(p)$  instead of  $V(p)$ .



# Feature detection and matching

*Input:* Features detected in each image.

*Output:* Initial sparse set of patches  $P$ .

$P \leftarrow \phi$ ;

For each image  $I$  with optical center  $O(I)$

For each feature  $f$  detected in  $I$

$F \leftarrow \{\text{Features satisfying the epipolar consistency}\}$ ;

Sort  $F$  in an increasing order of distance from  $O(I)$ ;

For each feature  $f' \in F$

// Test a patch candidate  $p$ ;

Initialize  $\mathbf{c}(p)$ ,  $\mathbf{n}(p)$  and  $R(p)$ ; // Eqs. (4, 5, 6)

Initialize  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

Refine  $\mathbf{c}(p)$  and  $\mathbf{n}(p)$ ; // (Sect.II-C)

Update  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

If  $|V^*(p)| < \gamma$

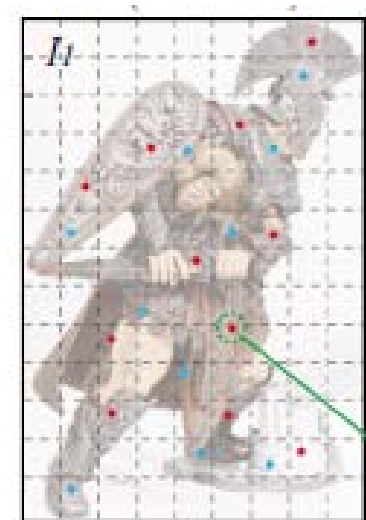
Go back to the innermost For loop (failure);

Add  $p$  to the corresponding  $Q_j(x,y)$  and  $Q_j^*(x,y)$ ;

Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;



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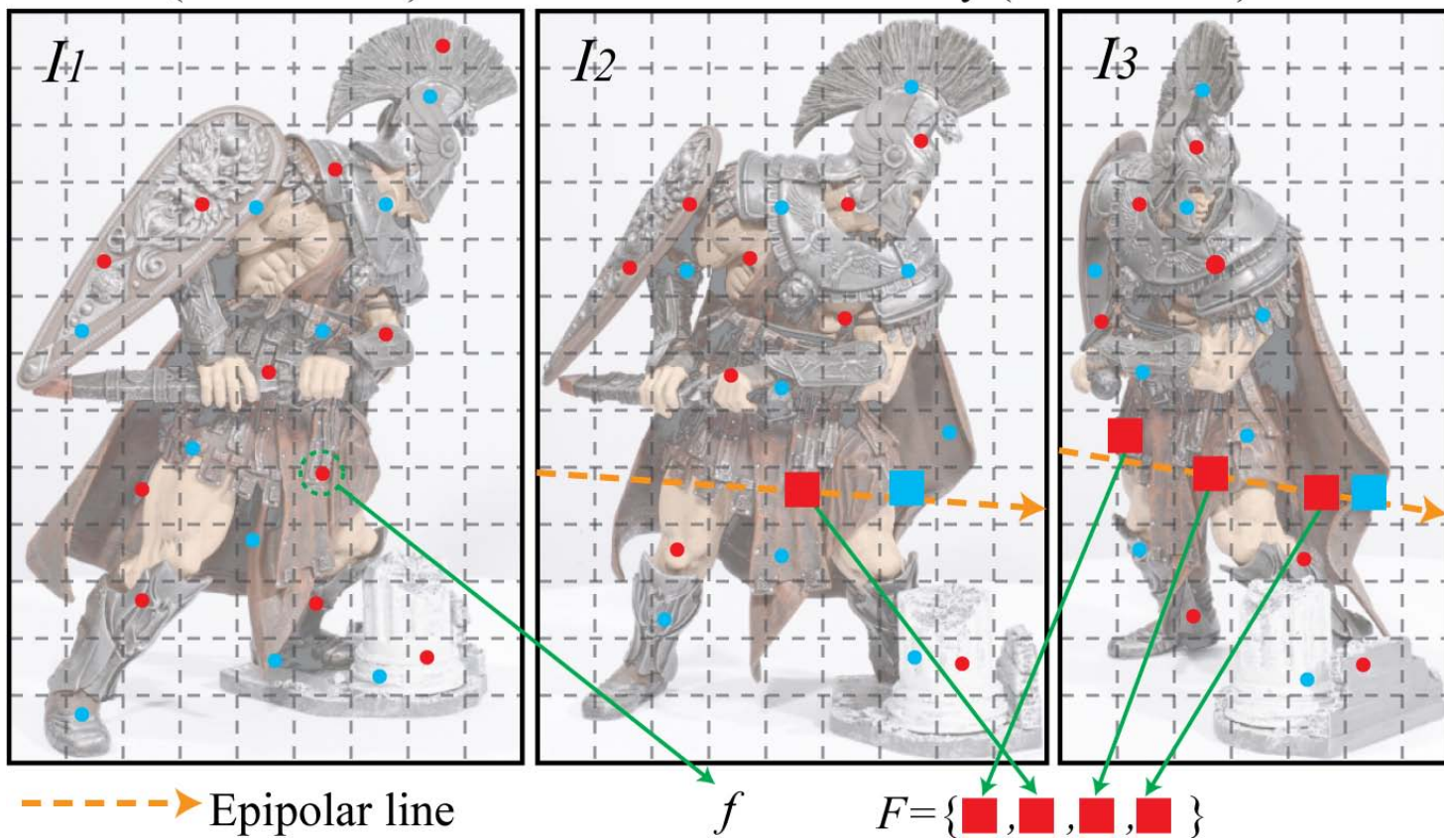
Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;

•/• Detected features  
(Harris/DoG)

■/■ Features satisfying epipolar  
consistency (Harris/DoG)





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Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;

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If  $|V^*(p)| < \gamma$

Go back to the innermost For loop (failure);

Add  $p$  to the corresponding  $Q_j(x,y)$  and  $Q_j^*(x,y)$ ;

Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;

$$\mathbf{c}(p) \leftarrow \{\text{Triangulation from } f \text{ and } f'\}, \quad (4)$$

$$\mathbf{n}(p) \leftarrow \frac{\overrightarrow{\mathbf{c}(p)O(I_i)}}{|\overrightarrow{\mathbf{c}(p)O(I_i)}|}, \quad (5)$$

$$R(p) \leftarrow I_i. \quad (6)$$

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*Input:* Features detected in each image.

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Refine  $\mathbf{c}(p)$  and  $\mathbf{n}(p)$ ; // (Sect.II-C)

Update  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

If  $|V^*(p)| < \gamma$

Go back to the innermost For loop (failure);

Add  $p$  to the corresponding  $Q_j(x,y)$  and  $Q_j^*(x,y)$ ;

Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;

# Feature detection and matching

*Input:* Features detected in each image.

*Output:* Initial sparse set of patches  $P$ .

$P \leftarrow \phi$ ;

For each image  $I$  with optical center  $O(I)$

For each feature  $f$  detected in  $I$

$F \leftarrow \{\text{Features satisfying the epipolar constraint}\}$

Sort  $F$  in an increasing order of distance from  $f$

For each feature  $f' \in F$

// Test a patch candidate  $p$ ;

Initialize  $\mathbf{c}(p)$ ,  $\mathbf{n}(p)$  and  $R(p)$ ; // Eqs. (4, 5, 6)

Initialize  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

Refine  $\mathbf{c}(p)$  and  $\mathbf{n}(p)$ ; // (Sect.II-C)

Update  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

If  $|V^*(p)| < \gamma$

Go back to the innermost For loop (failure);

Add  $p$  to the corresponding  $Q_j(x,y)$  and  $Q_j^*(x,y)$ ;

Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;

$$V^*(p) = \{I | I \in V(p), h(p, I, R(p)) \leq \alpha\}, \quad (2)$$

$$V(p) \leftarrow \{I | \mathbf{n}(p) \cdot \overrightarrow{\mathbf{c}(p)O(I)} / |\overrightarrow{\mathbf{c}(p)O(I)}| > \cos(\iota)\}. \quad (7)$$

# Feature detection and matching

*Input:* Features detected in each image.

*Output:* Initial sparse set of patches  $P$ .

$P \leftarrow \phi$ ;

For each image  $I$  with optical center  $O(I)$

For each feature  $f$  detected in  $I$

$F \leftarrow \{\text{Features satisfying the epipolar constraint}\}$

Sort  $F$  in an increasing order of distance from  $f$

For each feature  $f' \in F$

// Test a patch candidate  $p$ ;

Initialize  $\mathbf{c}(p)$ ,  $\mathbf{n}(p)$  and  $R(p)$ ; // Eqs. (4, 5)

Initialize  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

Refine  $\mathbf{c}(p)$  and  $\mathbf{n}(p)$ ; // (Sect.II-C)

Update  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

If  $|V^*(p)| < \gamma$

Go back to the innermost For loop (failure);

Add  $p$  to the corresponding  $Q_j(x,y)$  and  $Q_j^*(x,y)$ ;

Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;

**optimization** The geometric parameters,  $\mathbf{c}(p)$  and  $\mathbf{n}(p)$ , are optimized by simply minimizing the photometric

$$g^*(p) = \frac{1}{|V^*(p) \setminus R(p)|} \sum_{I \in V^*(p) \setminus R(p)} h(p, I, R(p)).$$

# Feature detection and matching

*Input:* Features detected in each image.

*Output:* Initial sparse set of patches  $P$ .

$P \leftarrow \phi$ ;

For each image  $I$  with optical center  $O(I)$

For each feature  $f$  detected in  $I$

$F \leftarrow \{\text{Features satisfying the epipolar consistency}\}$ ;

Sort  $F$  in an increasing order of distance from  $O(I)$ ;

For each feature  $f' \in F$

// Test a patch candidate  $p$ ;

Initialize  $\mathbf{c}(p)$ ,  $\mathbf{n}(p)$  and  $R(p)$ ; // Eqs. (4, 5, 6)

Initialize  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

Refine  $\mathbf{c}(p)$  and  $\mathbf{n}(p)$ ; // (Sect.II-C)

Update  $V(p)$  and  $V^*(p)$ ; // Eqs. (2, 7)

If  $|V^*(p)| < \gamma$

Go back to the innermost For loop (failure);

Add  $p$  to the corresponding  $Q_j(x, y)$  and  $Q_j^*(x, y)$ ;

Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;

# Feature detection and matching

*Input:* Features detected in each image.

*Output:* Initial sparse set of patches  $P$ .

$P \leftarrow \phi$ ;

For each image  $I$  with optical center  $O(I)$

For each feature  $f$  detected in  $I$

$F \leftarrow \{\text{Features satisfying the criteria}\}$

Sort  $F$  in an increasing order of  $\mathbf{c}(f)$

For each feature  $f' \in F$

// Test a patch candidate

Initialize  $\mathbf{c}(p)$ ,  $\mathbf{n}(p)$  and  $\mathbf{v}(p)$

Initialize  $V(p)$  and  $V^*(p)$

Refine  $\mathbf{c}(p)$  and  $\mathbf{n}(p)$ ; //

Update  $V(p)$  and  $V^*(p)$ ;

If  $|V^*(p)| < \gamma$

Go back to the innermost For loop (failure);

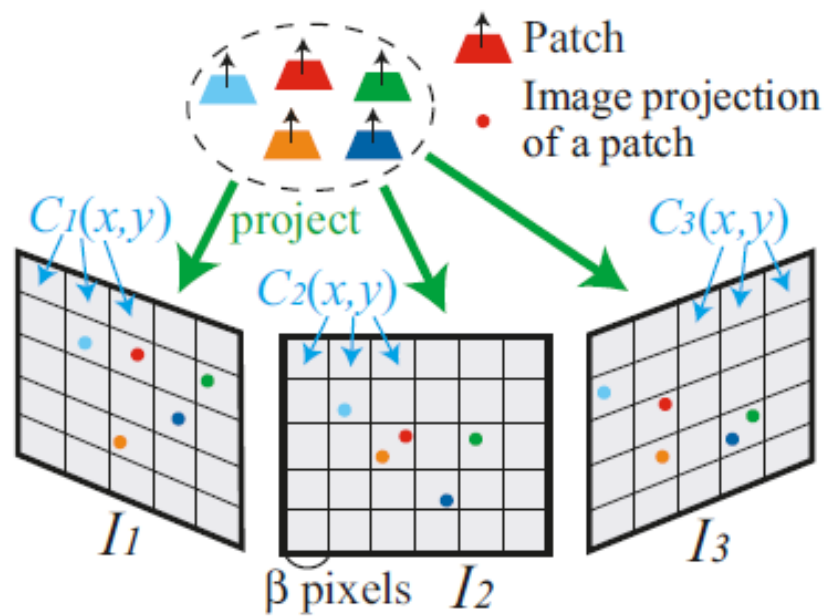
Add  $p$  to the corresponding  $Q_j(x,y)$  and  $Q_j^*(x,y)$ ;

Remove features from the cells where  $p$  was stored;

Add  $p$  to  $P$ ;

Exit innermost For loop;

$Q_j(x,y)$  and  $Q_j^*(x,y)$  Given a patch  $p$  and its visible images  $V(p)$ , we simply project  $p$  into each image in  $V(p)$  to identify the corresponding cell. Then, each cell  $C_i(x,y)$  remembers the set of patches  $Q_i(x,y)$  that project into it. Similarly, we use  $Q_i^*(x,y)$  to denote the patches that are obtained by the same procedure, but with  $V^*(p)$  instead of  $V(p)$ .





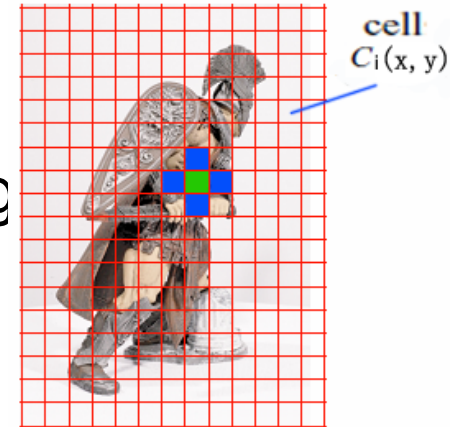
# Expansion

- Identifying Cells for expansion
- Expansion procedure

# Identifying Cells for expansion

Given a patch  $p$ , we initialize  $C(p)$  by collecting the neighboring image cell in its each visible image:

$$C(p) = \{C_i(x', y') | p \in Q_i(x, y), |x - x'| + |y - y'| = 1\}$$

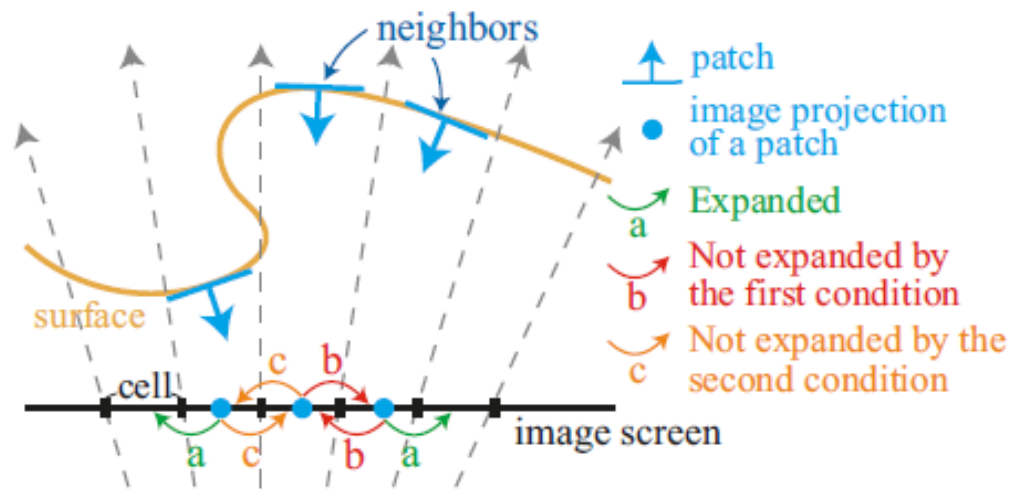


The expansion procedure is not performed for an image cell if

1. an image cell  $C_i(x', y') \in C(p)$  contains a patch  $p'$ , which is a neighbor of  $p$ ,  $C_i(x', y')$  is removed from the set of  $C(p)$

$$|(\mathbf{c}(p) - \mathbf{c}(p')) \cdot \mathbf{n}(p)| + |(\mathbf{c}(p) - \mathbf{c}(p')) \cdot \mathbf{n}(p')| < 2\rho_1$$

2.  $C_i(x', y')$  already contains a patch whose photometric discrepancy score associated with  $I_i$  is better than the threshold  $\alpha$ .



# Expansion procedure

*Input:* Patches  $P$  from the feature matching step.

*Output:* Expanded set of reconstructed patches.

*While*  $P$  is not empty

    Pick and remove a patch  $p$  from  $P$ ;

*For each* image cell  $C_i(x,y)$  containing  $p$

        Collect a set  $C$  of image cells for expansion;

*For each* cell  $C_i(x',y')$  in  $C$

            // Create a new patch candidate  $p'$

$\mathbf{n}(p') \leftarrow \mathbf{n}(p)$ ,  $R(p') \leftarrow R(p)$ ,  $V(p') \leftarrow V^*(p)$ ;

            Update  $V^*(p)$ ; // Eq. (2)

            Refine  $\mathbf{c}(p')$  and  $\mathbf{n}(p')$ ; // (Sect.II-C)

            Add visible images (a depth-map test) to  $V(p')$ ;

            Update  $V^*(p')$ ; // Eq. (2)

            If  $|V^*(p')| < \gamma$

                Go back to *For*-loop (failure);

            Add  $p'$  to  $P$ ;

            Add  $p'$  to corresponding  $Q_j(x,y)$  and  $Q_j^*(x,y)$ ;

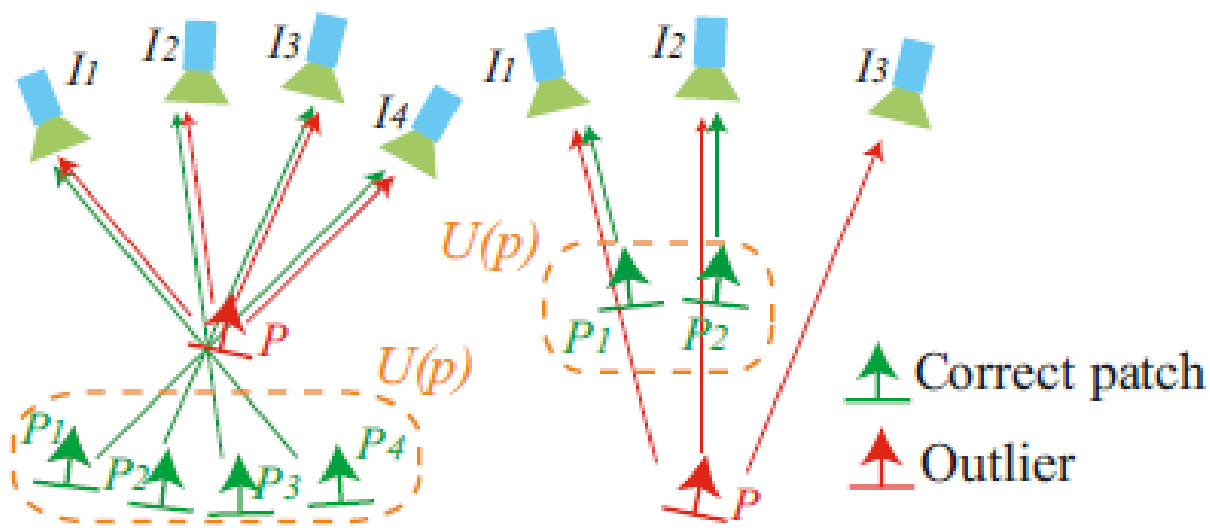
# Filtering

A patch  $p$  will be removed in the following three situations:

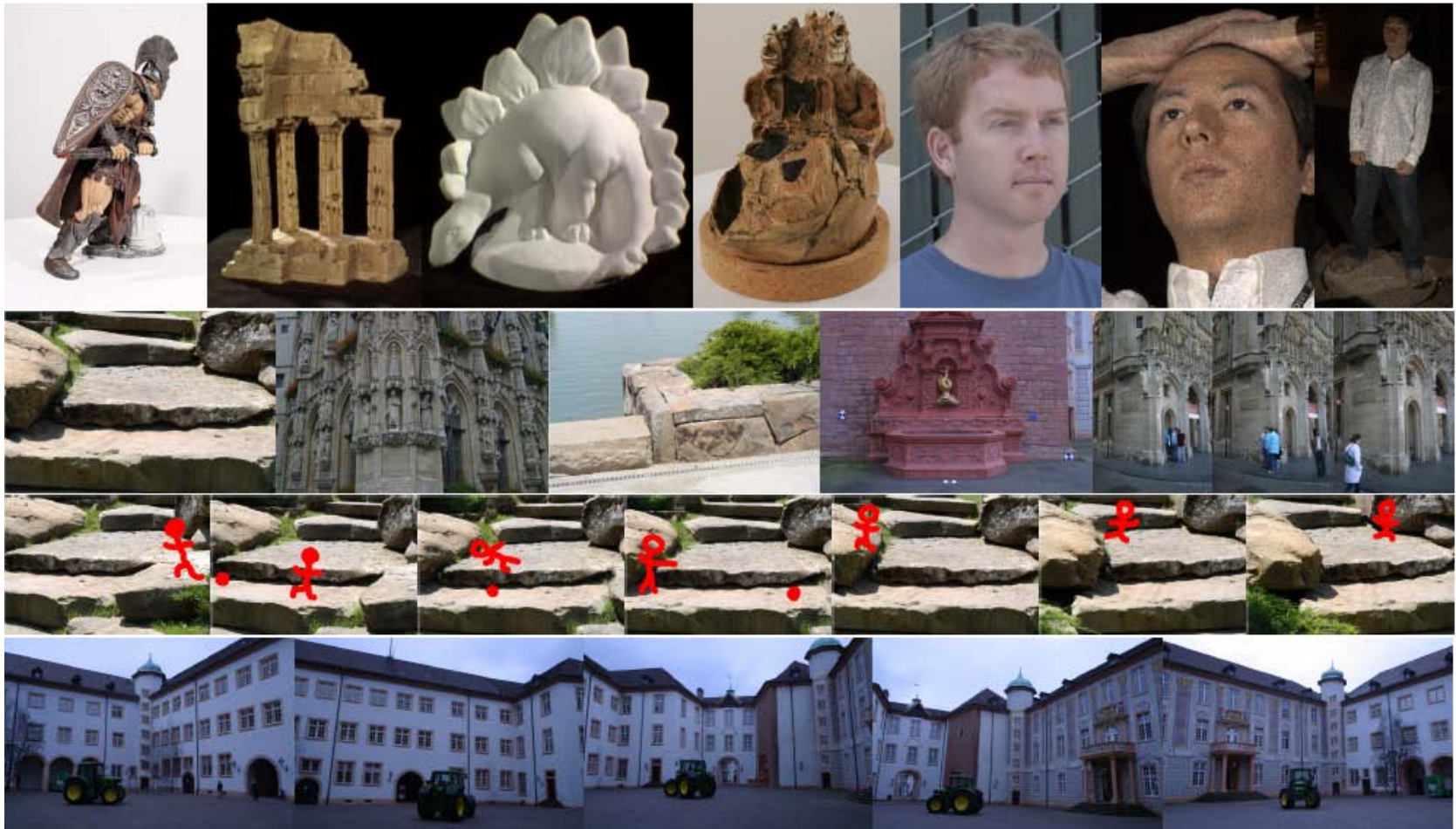
1.  $p$  and  $p'$  are not neighbors, but are stored in the same cell of one of the images where  $p$  is visible. Then  $p$  is filtered out as an outlier if the following inequality holds

$$|V^*(p)|(1 - g^*(p)) < \sum_{p_i \in U(p)} 1 - g^*(p_i).$$

2. If the number of images in  $V^*(p)$  according to depth-map test is less than  $\gamma$ .
3. If the proportion of patches that are neighbors of  $p$  in  $V(p)$  is lower than 0.25.



# Some results in the paper









# test

