

High-precision, consistent EKF-based visual-inertial odometry

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Introduction

- What is visual-inertial odometry (VIO) ?

The problem of motion tracking in unknown environments using visual and inertial sensors. (no mapping)

Introduction

- Main algorithms

1. Extended Kalman filter (EKF)-based methods

2. Methods utilizing iterative minimization over a window of states. (more accurate, but high computational cost)

Introduction

- Two families of EKF-based VIO estimators:
 1. simultaneous localization and mapping. (EKF-SLAM)
 2. sliding-window algorithms. (MSCKF)

MSCKF is more accurate and faster.

Introduction

- But MSCKF is not good enough.

Both EKF-SLAM and MSCKF are inconsistent.

Introduction

What is consistency?

A recursive estimator is consistent when the **estimation errors** are **zero-mean** and have **covariance matrix equal to that reported by the estimator**.

Introduction

- Main cause of inconsistency in the MSCKF (and EKF-SLAM):

The uncertainty of all states are underestimated.

Introduction

- EKF is faster, iterative-minimization algorithms is more accurate.
- Ideally, one would like to obtain accuracy similar to, or better than, that of iterative-minimization algorithms, but at the computational cost of an EKF algorithm.
- In this paper, we show how this can be achieved.

Related Work

- EKF-SLAM
- MSCKF

In this paper, we compare the MSCKF's accuracy and consistency to those of EKF-SLAM methods, and show that the MSCKF outperforms EKF-SLAM in these respects as well.

Related Work

A key contribution of this work is the analysis and improvement of the consistency of EKF-based vision aided inertial navigation.

Past work on the consistency of 3D vision-based localization has primarily focused on the parameterization of feature positions.

Related Work

(Civera et al., 2008) showed that the Cartesian-coordinate (XYZ) parametrization results in severely non-Gaussian probability density functions (pdfs) for the features, and degrades accuracy and consistency. Therefore, an inverse-depth feature parametrization was proposed.

Sola (2010) proposed an anchored homogeneous feature parametrization that was shown to further improve the filter's consistency

Related Work

In our work, we compare all of the above parameterizations in VIO and show that, while the parameterization of (Sola, 2010) yields superior results to the alternatives, its performance is still worse than that of the MSCKF algorithm.

Related Work

This work is based on recent work examining the relationship between the **observability properties** of the EKF's linearized system model and the filter's **consistency**.



Related Work

What is observability properties ?

In control theory, observability is a measure for how well internal states of a system can be inferred by knowledge of its external outputs.

Introduced by Kalman.

Related Work

Recent works showed that, due to the way in which Jacobians are computed in the EKF, the robot's orientation appears to be observable in the linearized system model, while it is not in the actual, nonlinear system.

As a result of this mismatch, the filter produces too small estimates for the uncertainty of the orientation estimates, and becomes inconsistent.

Mismatch in observability



Inconsistent

EKF-based VIO

- IMU state

{G} global coordinate frame

{I} IMU coordinate frame

$$\mathbf{x}_{I_\ell} = \begin{bmatrix} I_\ell \bar{\mathbf{q}}^T & \mathbf{p}_\ell^T & \mathbf{v}_\ell^T & \mathbf{b}_{g_\ell}^T & \mathbf{b}_{a_\ell}^T \end{bmatrix}^T$$

Time step

EKF-based VIO

- EKF introduction

general nonlinear models

$$\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u}) + \mathbf{w}$$

$$\mathbf{z} = h(\mathbf{x}) + \mathbf{n}$$

linearized version of the discrete-time model

$$\tilde{\mathbf{x}}_{l+1} \simeq \mathbf{\Phi}_l \tilde{\mathbf{x}}_l + \mathbf{w}_l$$

$$\tilde{\mathbf{r}}_l \simeq \mathbf{H}_l \tilde{\mathbf{x}}_l + \mathbf{n}_l$$

Used for covariance propagation and update, gain computation

EKF-based VIO

- IMU error state

$$\tilde{\mathbf{x}}_I = \left[\begin{array}{c} G\tilde{\boldsymbol{\theta}}^T \\ G\tilde{\mathbf{p}}^T \\ G\tilde{\mathbf{v}}^T \\ \tilde{\mathbf{b}}_g^T \\ \tilde{\mathbf{b}}_a^T \end{array} \right]^T \quad 15 \times 1$$

from quaternion

EKF-based VIO

- IMU error state

IMU error-state transition matrix


$$\tilde{\mathbf{x}}_{I_{\ell+1}|\ell} \simeq \mathbf{\Phi}_{I_{\ell}} \tilde{\mathbf{x}}_{I_{\ell}|\ell} + \mathbf{w}_{d_{\ell}}$$

EKF-based VIO

- A. EKF-SLAM

$$\mathbf{x}_\ell = \left[\mathbf{x}_{I_\ell}^T \quad \mathbf{f}_1^T \quad \dots \quad \mathbf{f}_{n_\ell}^T \right]^T$$

Feature position



EKF-based VIO

- A. EKF-SLAM

Measurement model

feature i in camera frame

$$\mathbf{z}_{il} = \mathbf{h}(\mathbf{x}_{I_l}, \mathbf{f}_i) + \mathbf{n}_{il} = \begin{bmatrix} c_{lx_{f_i}} \\ c_{lz_{f_i}} \\ c_{ly_{f_i}} \\ c_{lz_{f_i}} \end{bmatrix} + \mathbf{n}_{il}$$

Residual

$$\begin{aligned} \mathbf{r}_{il} &= \mathbf{z}_{il} - \mathbf{h}(\hat{\mathbf{x}}_{I_{l|l-1}}, \hat{\mathbf{f}}_{i_{l|l-1}}) \\ &\simeq \mathbf{H}_{il}(\hat{\mathbf{x}}_{l|l-1}) \tilde{\mathbf{x}}_{l|l-1} + \mathbf{n}_{il} \end{aligned}$$

Linearization point

EKF-based VIO

- B. MSCKF

$$\mathbf{x}_\ell = \left[\mathbf{x}_{I_\ell}^T \quad \boldsymbol{\pi}_{\ell-1}^T \quad \boldsymbol{\pi}_{\ell-2}^T \quad \cdots \quad \boldsymbol{\pi}_{\ell-N}^T \right]^T$$

Pose



EKF-base

- B. MSCKF

Algorithm 1 Multi-state-constraint Kalman filter
(MSCKF)

Propagation: Propagate state vector and covariance matrix using IMU readings.

Update: when a new image is recorded,

- State augmentation: augment the state vector and state covariance matrix with the current IMU position and orientation.
 - Image processing: extract corner features and perform feature matching.
 - Update: for each feature whose track is complete, compute \mathbf{r}_i^o and \mathbf{H}_i^o , and perform the Mahalanobis test. Use all features that pass the test for an EKF update.
 - State management: remove from the state vector those IMU states for which all associated features have been processed.
-

EKF-based VIO

- B. MSCKF

We consider the case where the feature f_i , observed from the N poses in the MSCKF state vector, is used for an update at time step l .

Computed by triangulation

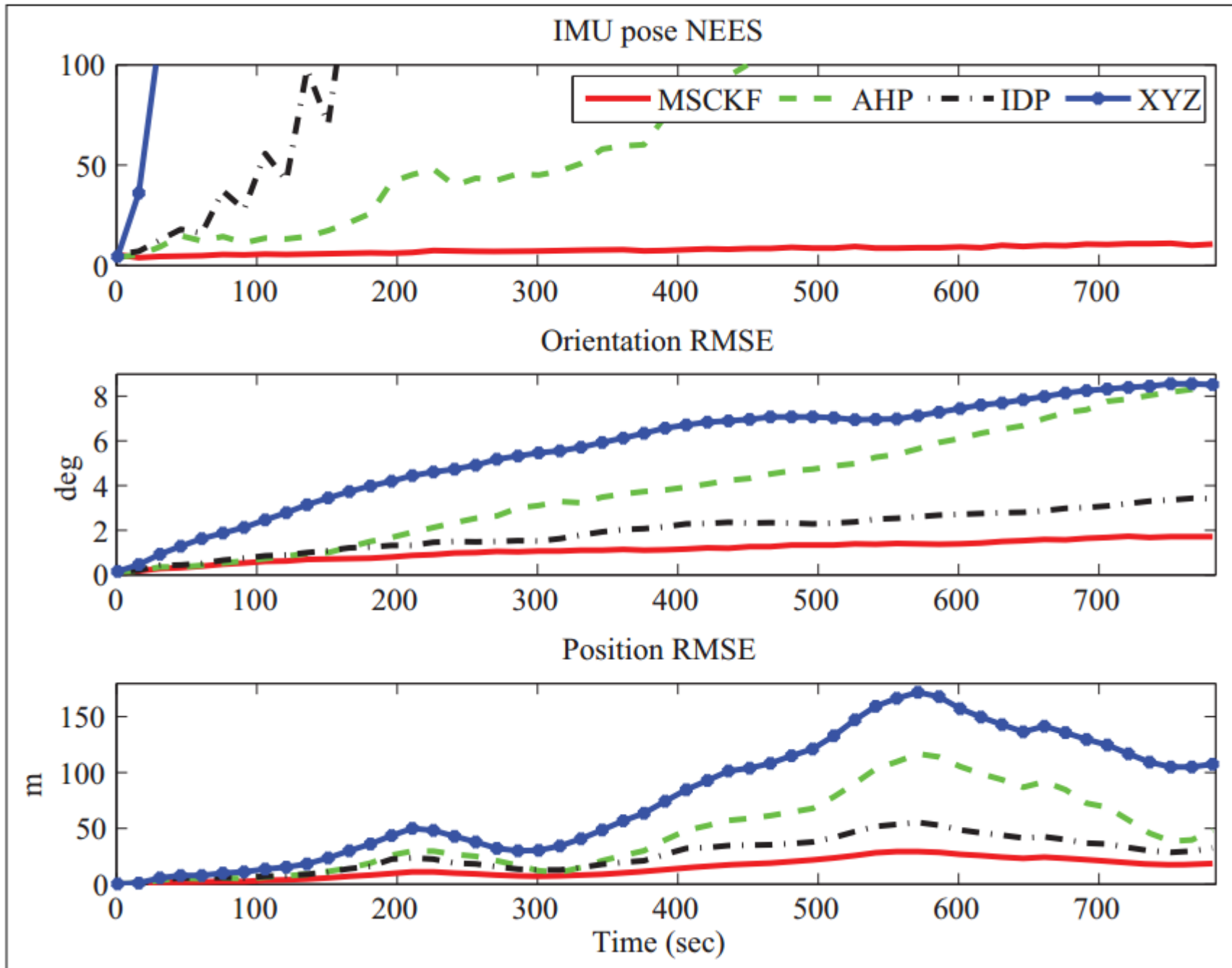
$$\mathbf{r}_{ij} = \mathbf{z}_{ij} - \mathbf{h}(\hat{\boldsymbol{\pi}}_{j|l-1}, \boxed{{}^G \hat{\mathbf{p}}_{f_i}})$$
$$\simeq \mathbf{H}_{\pi_{ij}}(\hat{\boldsymbol{\pi}}_{j|l-1}, \boxed{{}^G \hat{\mathbf{p}}_{f_i}}) \tilde{\boldsymbol{\pi}}_{j|l-1} + \mathbf{H}_{f_{ij}}(\hat{\boldsymbol{\pi}}_{j|l-1}, \boxed{{}^G \hat{\mathbf{p}}_{f_i}}) \boxed{{}^G \tilde{\mathbf{p}}_{f_i}}$$

+ \mathbf{n}_{ij}

Linearization point

EKF

• Cor



EKF-based VIO

We attribute to two main reasons

1. First, all EKF-SLAM algorithms assume that the errors of the IMU state and feature positions are jointly Gaussian at each time step.

2. MSCKF employs a “delayed linearization” approach: it processes each feature only when all of its measurements become available.

This means that more accurate feature estimates are used in computing Jacobians, leading to more precise calculation of the Kalman gain and state corrections, and ultimately better accuracy.

EKF consistency and relation to observability

- EKF consistency and relation to observability.

general nonlinear models

$$\begin{aligned}\dot{\mathbf{x}} &= f(\mathbf{x}, \mathbf{u}) + \mathbf{w} \\ \mathbf{z} &= h(\mathbf{x}) + \mathbf{n}\end{aligned}$$

linearized version of the discrete-time model

$$\begin{aligned}\tilde{\mathbf{x}}_{l+1} &\simeq \mathbf{\Phi}_l \tilde{\mathbf{x}}_l + \mathbf{w}_l \\ \tilde{\mathbf{r}}_l &\simeq \mathbf{H}_l \tilde{\mathbf{x}}_l + \mathbf{n}_l\end{aligned}$$

Used for covariance propagation and update, gain computation

EKF consistency and relation to observability

- It has been proved that when a camera/IMU system moves in a general trajectory, in an environment with a known gravitational acceleration but no known features, four degrees of freedom are unobservable:
 - (i) three of them corresponding to the global position
 - (ii) one corresponding to the rotation about the gravity vector (i.e. the yaw).

EKF consistency and relation to observability

- How to know linearized model's unobservable subspace ?

studying the **nullspace of the observability matrix**

$$\mathcal{O} \triangleq \begin{bmatrix} \mathbf{H}_k \\ \mathbf{H}_{k+1} \Phi_k \\ \vdots \\ \mathbf{H}_{k+m} \Phi_{k+m-1} \cdots \Phi_k \end{bmatrix}$$

EKF consistency and relation to observability

Φ

- before proceeding with the observability analysis, we must derive an expression for error transition matrix.

$$\Phi_{I_e}(\hat{\mathbf{x}}_{I_{e+1}}, \hat{\mathbf{x}}_{I_e}) = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_3 & \mathbf{0}_3 \\ \Phi_{\text{pq}}(\hat{\mathbf{x}}_{I_{e+1}}, \hat{\mathbf{x}}_{I_e}) & \mathbf{I}_3 & \Delta t \mathbf{I}_3 \\ \Phi_{\text{vq}}(\hat{\mathbf{x}}_{I_{e+1}}, \hat{\mathbf{x}}_{I_e}) & \mathbf{0}_3 & \mathbf{I}_3 \end{bmatrix}$$

$$\Phi_{\text{pq}}(\hat{\mathbf{x}}_{I_{e+1}}, \hat{\mathbf{x}}_{I_e})$$

$$= -\left[({}^G \hat{\mathbf{p}}_{e+1} - {}^G \hat{\mathbf{p}}_e - {}^G \hat{\mathbf{v}}_e \Delta t - \frac{1}{2} \mathbf{g} \Delta t^2) \times \right]$$

$$\Phi_{\text{vq}}(\hat{\mathbf{x}}_{I_{e+1}}, \hat{\mathbf{x}}_{I_e}) = -\left[({}^G \hat{\mathbf{v}}_{e+1} - {}^G \hat{\mathbf{v}}_e - \mathbf{g} \Delta t) \times \right]$$

Observability properties of the MSCKF system model

H

- Given a linear (or, equivalently, a linearized) model, the EKF-SLAM and MSCKF measurement equations are equivalent.
- This means that we can study the observability of the MSCKF's linearized model by studying the EKF-SLAM linearized model, but using the MSCKF's linearization points.

Observability properties of the MSCKF system model

- if feature i is processed at time step α_i+1

up to α_i computed via triangulation

$$\mathbf{H}_{il}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{fi})$$

$$= [\mathbf{H}_{Iil}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{fi}) \mathbf{0} \cdots \mathbf{H}_{fil}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{fi}) \cdots \mathbf{0}]$$

MSCKF's linearization point

$$\mathbf{H}_{fil}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{fi}) = \mathbf{J}_{il}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{fi}) {}^C\mathbf{R} {}^I\mathbf{R} \hat{\mathbf{R}}_{\ell|\alpha_i}$$

$$\mathbf{H}_{Iil}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{fi}) = \mathbf{H}_{fil}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{fi})$$

$$[\mathbf{L}({}^G\hat{\mathbf{p}}_{fi} - {}^G\hat{\mathbf{p}}_{\ell|\alpha_i}) \times] \quad -\mathbf{I}_3 \quad \mathbf{0}_3$$

$$\mathbf{J}_{il}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{fi}) = \frac{1}{c_{\ell\hat{z}_{fi}}} \begin{bmatrix} 1 & 0 & \frac{-c_{\ell\hat{x}_{fi}}}{c_{\ell\hat{z}_{fi}}} \\ 0 & 1 & \frac{-c_{\ell\hat{y}_{fi}}}{c_{\ell\hat{z}_{fi}}} \end{bmatrix}$$

EKF consistency and relation to observability

- “Ideal” observability matrix

use true value to compute observability matrix

$$\check{O}_{il} = \check{M}_{il} [\check{\Gamma}_{il} \quad -\mathbf{I}_3 \quad -\Delta t_l \mathbf{I}_3 \quad \mathbf{0}_3 \quad \cdots \quad \mathbf{I}_3 \quad \cdots \quad \mathbf{0}_3]$$

$$\check{M}_{il} = \check{J}_{il} \mathbf{C}_I \mathbf{R} \mathbf{R}_l$$

$$\check{\Gamma}_{il} = \left[\left({}^G \mathbf{p}_{f_i} - {}^G \mathbf{p}_k - {}^G \mathbf{v}_k \Delta t_l - \frac{1}{2} \mathbf{g} \Delta t_l^2 \right) \times \right]$$

$$\mathbf{N} = \begin{bmatrix} \mathbf{0}_3 & \mathbf{g} \\ \mathbf{I}_3 & -\left[{}^G \mathbf{p}_k \times \right] \mathbf{g} \\ \mathbf{0}_3 & -\left[{}^G \mathbf{v}_k \times \right] \mathbf{g} \\ \mathbf{I}_3 & -\left[{}^G \mathbf{p}_{f_1} \times \right] \mathbf{g} \\ \mathbf{I}_3 & -\left[{}^G \mathbf{p}_{f_2} \times \right] \mathbf{g} \\ \vdots & \vdots \\ \mathbf{I}_3 & -\left[{}^G \mathbf{p}_{f_N} \times \right] \mathbf{g} \end{bmatrix}$$

Nullspace of observability matrix is 4.

EKF consistency and relation to observability

- MSCKF observability matrix

$$\mathbf{O}_{il} = \mathbf{M}_{il} [\mathbf{\Gamma}_{il} + \boxed{\Delta\mathbf{\Gamma}_{il}} \quad -\mathbf{I}_3 \quad -\Delta t_l \mathbf{I}_3 \quad \mathbf{0}_3 \quad \cdots \quad \mathbf{I}_3 \quad \cdots \quad \mathbf{0}_3]$$

$$\Delta\mathbf{\Gamma}_{il} = [{}^G\hat{\mathbf{p}}_{\ell|\ell-1} - {}^G\hat{\mathbf{p}}_{\ell|\alpha_i} \times] + \sum_{j=k+1}^{\ell-1} (\mathbf{E}_{\mathbf{p}}^j + \sum_{s=k+1}^j \mathbf{E}_{\mathbf{v}}^s \Delta t)$$

$$\mathbf{E}_{\mathbf{p}}^j = [{}^G\hat{\mathbf{p}}_{j|j-1} - {}^G\hat{\mathbf{p}}_{j|j} \times]$$

$$\mathbf{E}_{\mathbf{v}}^j = [{}^G\hat{\mathbf{v}}_{j|j-1} - {}^G\hat{\mathbf{v}}_{j|j} \times]$$

Nullspace of observability matrix
Is 3 now.

Mismatch in observability



Inconsistent

MSCKF 2.0

- Only use the first estimates

$$\mathbf{H}_{f_{i\ell}}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{f_i}) = \mathbf{J}_{i\ell}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{f_i}) {}^C\mathbf{R} {}^R\hat{\mathbf{R}}_{\ell|\alpha_i}$$

$$\mathbf{H}_{I_{i\ell}}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{f_i}) = \mathbf{H}_{f_{i\ell}}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{f_i})$$

$$[\mathbf{L}({}^G\hat{\mathbf{p}}_{f_i} - {}^G\hat{\mathbf{p}}_{\ell|\alpha_i}) \times] \quad -\mathbf{I}_3 \quad \mathbf{0}_3$$

$$\mathbf{J}_{i\ell}(\hat{\boldsymbol{\pi}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{f_i}) = \frac{1}{c_{\ell}\hat{z}_{f_i}} \begin{bmatrix} 1 & 0 & \frac{-c_{\ell}\hat{x}_{f_i}}{c_{\ell}\hat{z}_{f_i}} \\ 0 & 1 & \frac{-c_{\ell}\hat{y}_{f_i}}{c_{\ell}\hat{z}_{f_i}} \end{bmatrix}$$

$$\Delta\boldsymbol{\Gamma}_{i\ell} = [\mathbf{L}({}^G\hat{\mathbf{p}}_{\ell|\ell-1} - \boxed{{}^G\hat{\mathbf{p}}_{\ell|\alpha_i}}) \times] + \sum_{j=k+1}^{\ell-1} (\mathbf{E}_{\mathbf{p}}^j + \sum_{s=k+1}^j \mathbf{E}_{\mathbf{v}}^s \Delta t)$$

$$\mathbf{E}_{\mathbf{p}}^j = [\mathbf{L}({}^G\hat{\mathbf{p}}_{j|j-1} - \boxed{{}^G\hat{\mathbf{p}}_{j|j}}) \times]$$

$$\mathbf{E}_{\mathbf{v}}^j = [\mathbf{L}({}^G\hat{\mathbf{v}}_{j|j-1} - \boxed{{}^G\hat{\mathbf{v}}_{j|j}}) \times]$$

$$\Phi_{\ell}(\hat{\mathbf{x}}_{I_{\ell+1}|\ell}, \hat{\mathbf{x}}_{I_{\ell}|\ell}) = \begin{bmatrix} \Phi_{I_{\ell}}(\hat{\mathbf{x}}_{I_{\ell+1}|\ell}, \hat{\mathbf{x}}_{I_{\ell}|\ell}) & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{3M \times 3M} \end{bmatrix}$$

MSCKF 2.0

- Compute the IMU error-state transition matrix as

$$\Phi_{I_\ell}^* (\hat{\mathbf{x}}_{I_{\ell+1}|\ell}, \hat{\mathbf{x}}_{I_\ell|\ell-1})$$

- Calculate measurement Jacobians as

$$\mathbf{H}_{f_{i\ell}}^* = \mathbf{J}_{i\ell}(\hat{\mathbf{x}}_{\ell|\alpha_i}, {}^G\hat{\mathbf{p}}_{f_i}) \begin{matrix} C \\ I \end{matrix} \mathbf{R} \hat{\mathbf{R}}_{\ell|\alpha_i}$$

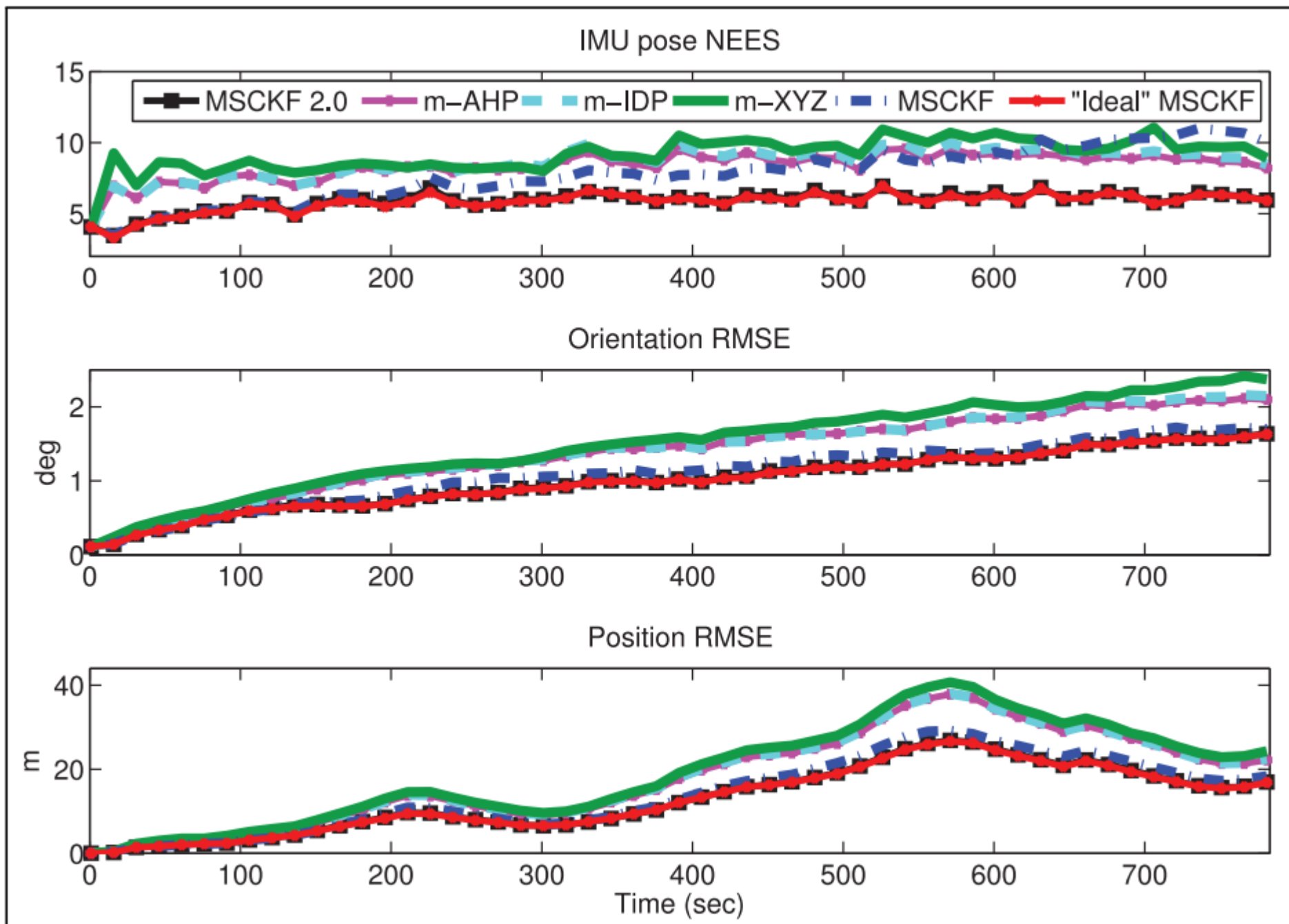
$$\mathbf{H}_{I_{i\ell}}^* = \mathbf{H}_{f_{i\ell}}^* \left[\lfloor ({}^G\hat{\mathbf{p}}_{f_i} - {}^G\hat{\mathbf{p}}_{\ell|\ell-1}) \times \rfloor \quad -\mathbf{I}_3 \quad \mathbf{0}_3 \right]$$



Simulation results

- The idea of using the first estimates of all states to ensure the correct observability properties of the linearized system model can also be employed for EKF-SLAM VIO.
- The resulting EKF-SLAM algorithms outperform the standard ones, yet cannot reach the accuracy or consistency of the MSCKF 2.0.

Sim



Simulation results

- What can we infer ?

These results show that **enforcing the correct observability properties of the linearized system is crucially important** for the performance of all EKF-based VIO methods.

MSCKF copes better with nonlinearities by not making Gaussian assumptions about the feature pdfs.

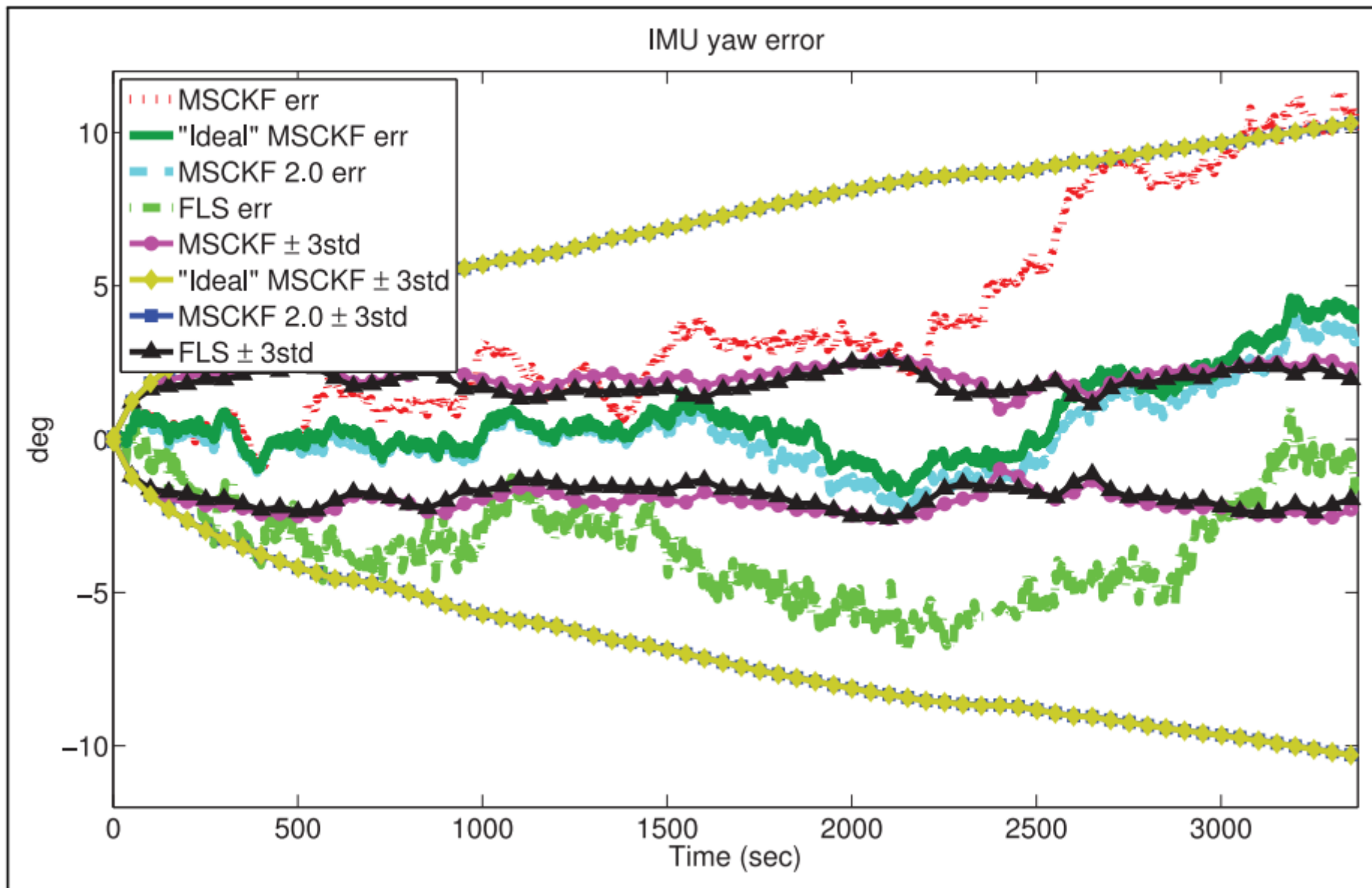
Simulation results

- What can we infer ?

This indicates that, as long as the correct observability properties are ensured, using slightly less accurate linearization points in computing the Jacobians does not significantly degrade the estimation performance.

Sim

- Col



Real-world experiment

- Configuration

Area:	streets of Riverside, CA
IMU:	Xsens MTi-G unit , 100 Hz
Camera:	PointGrey Bumblebee2 , 20 Hz
Feature extraction :	Shi-Tomasi algorithm
Ground truth:	GPS-INS estimate of the trajectory
Period:	37 minutes, approximately 21.5 km.

Real-world experiment

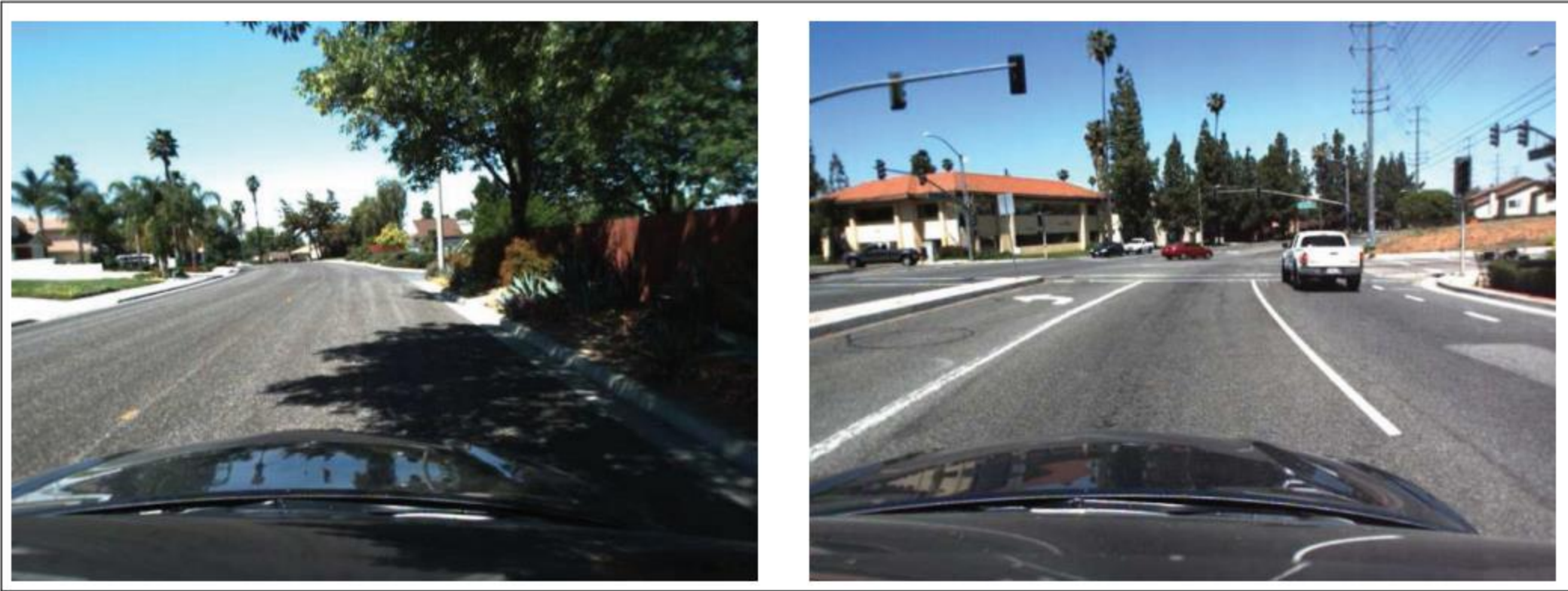


Fig. 7. Sample images recorded during the experiment.

Real-world experiment



Real-world experiment

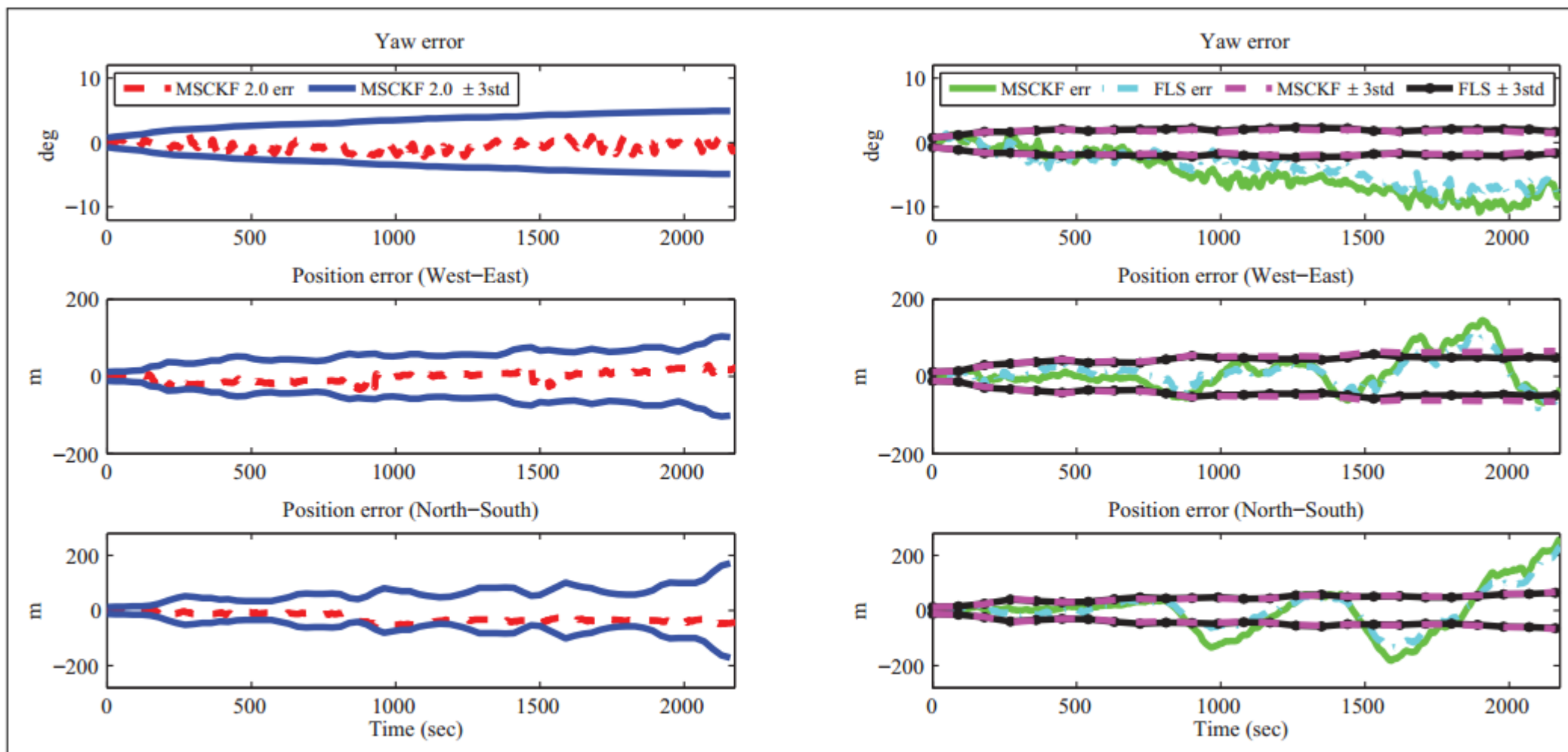


Fig. 6. Estimation errors for the three approaches. The left plots are the results for the MSCKF 2.0, and the right plots for the MSCKF and FLS.

Conclusion

- We showed that the MSCKF algorithm attains better accuracy and consistency than EKF-based SLAM due to its less strict probabilistic assumptions and delayed linearization.
- In addition, we performed a rigorous study of the consistency properties of EKF-based VIO algorithms, and the proposed MSCKF 2.0 algorithm is capable of performing long-term, high-precision, consistent VIO in real time.

工作安排

- 实现一个MSCKF 系统
- 探索如何进行mapping

Q&A